Theory of Particle Acceleration at Collisionless Shocks
Les Houches 2017

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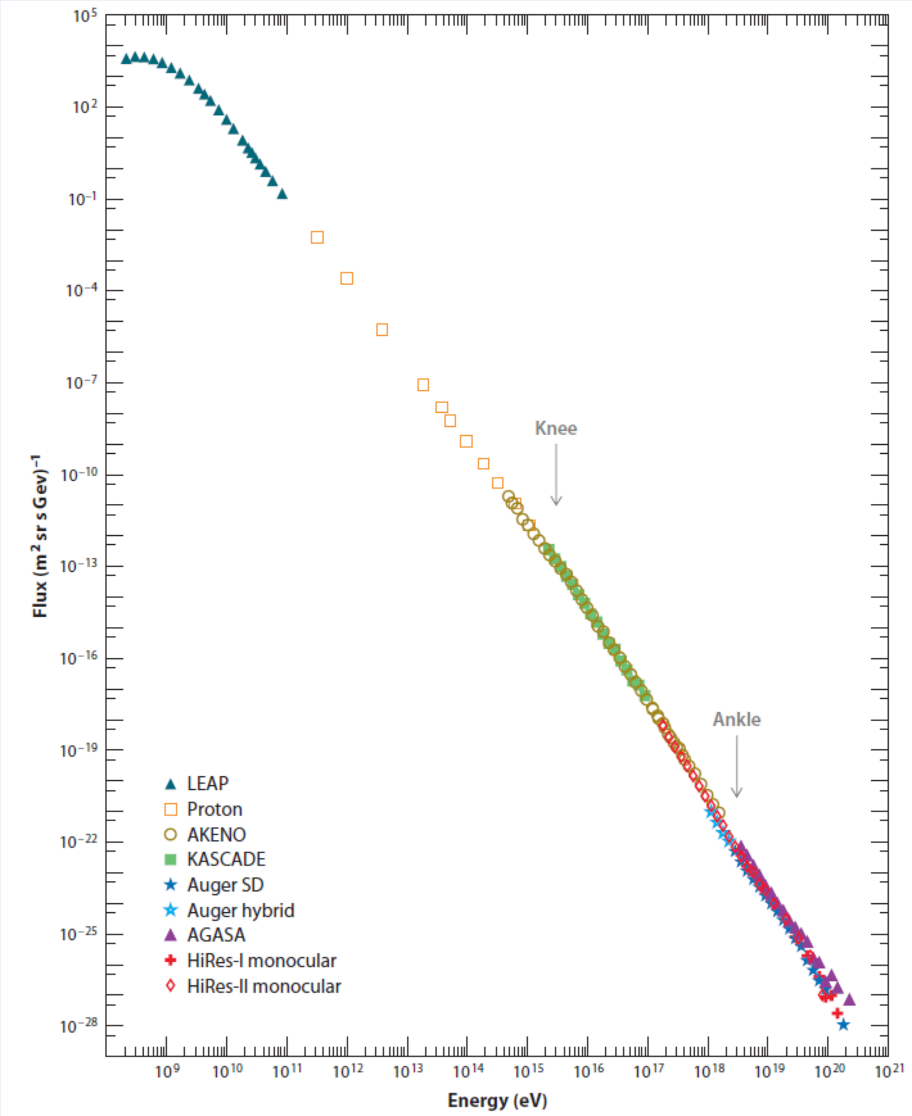
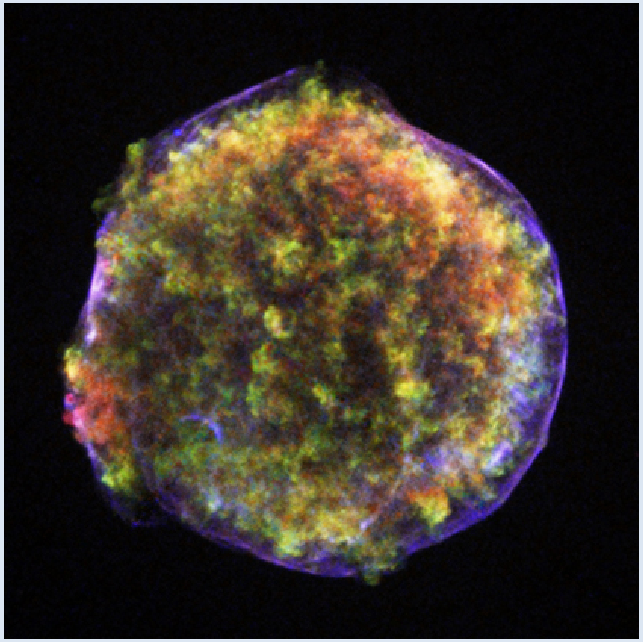
With thanks to Manfred Scholer

OUTLINE

- Motivation
- Collisionless shocks – the HOW? And WHY?
    - MHD conservation relations & discontinuities
    - Shock parameters
    - Structure of collisionless shocks
- Particle acceleration at shocks
    - Shock drift acceleration
    - Diffusive shock acceleration
    - Simulating ion injection
    - Global aspects of shock acceleration

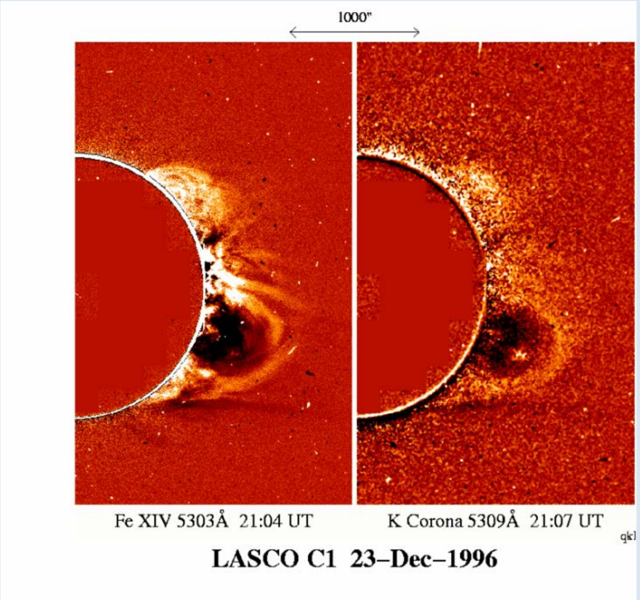
Cosmic Rays



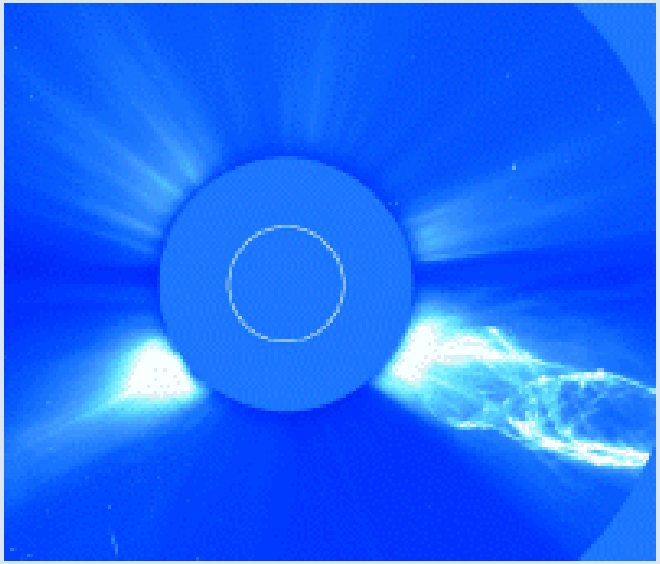


Beatty & Westerhoff 2009

Interplanetary shocks

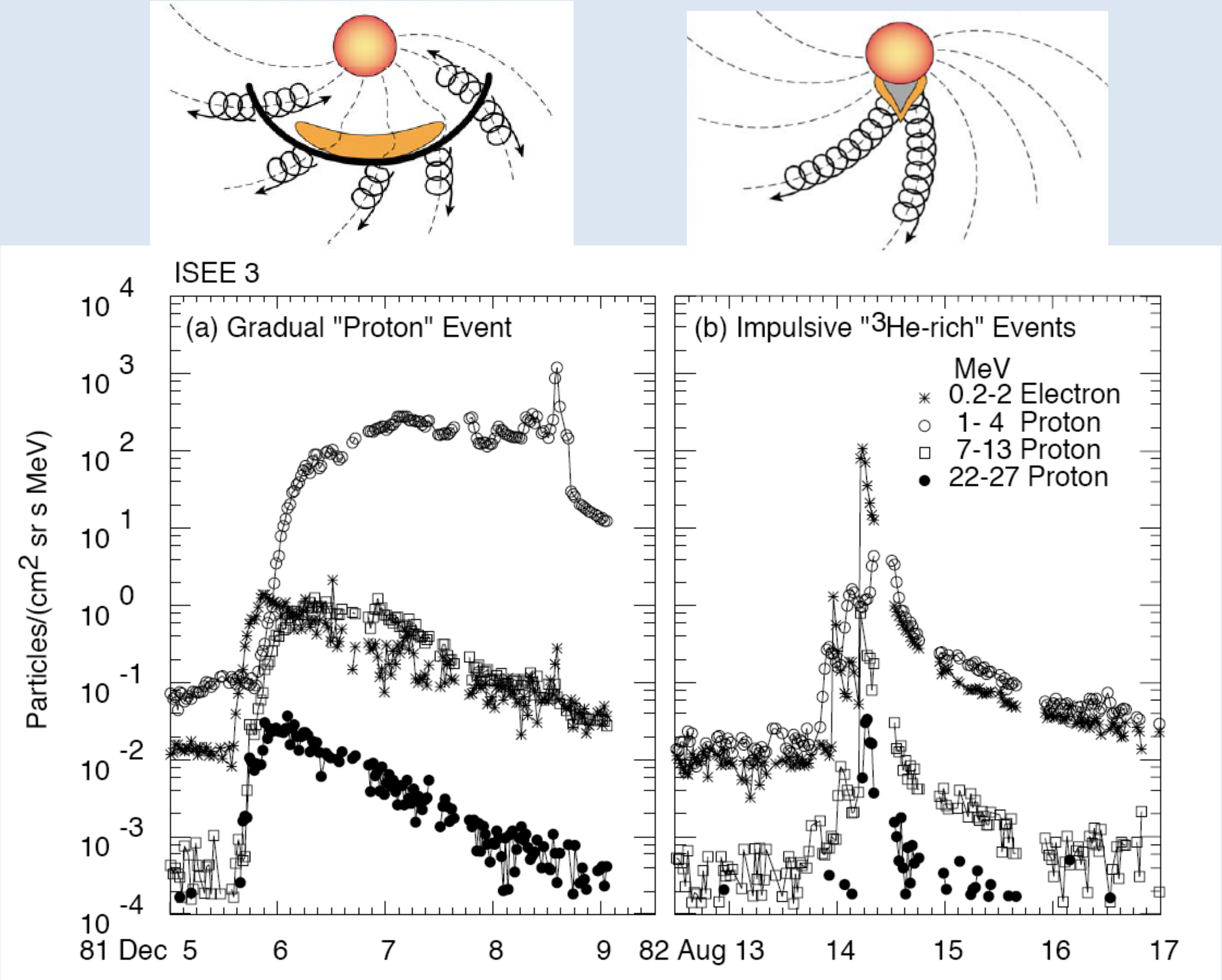


Coronal Mass Ejection (SOHO-LASCO) in forbidden Fe line



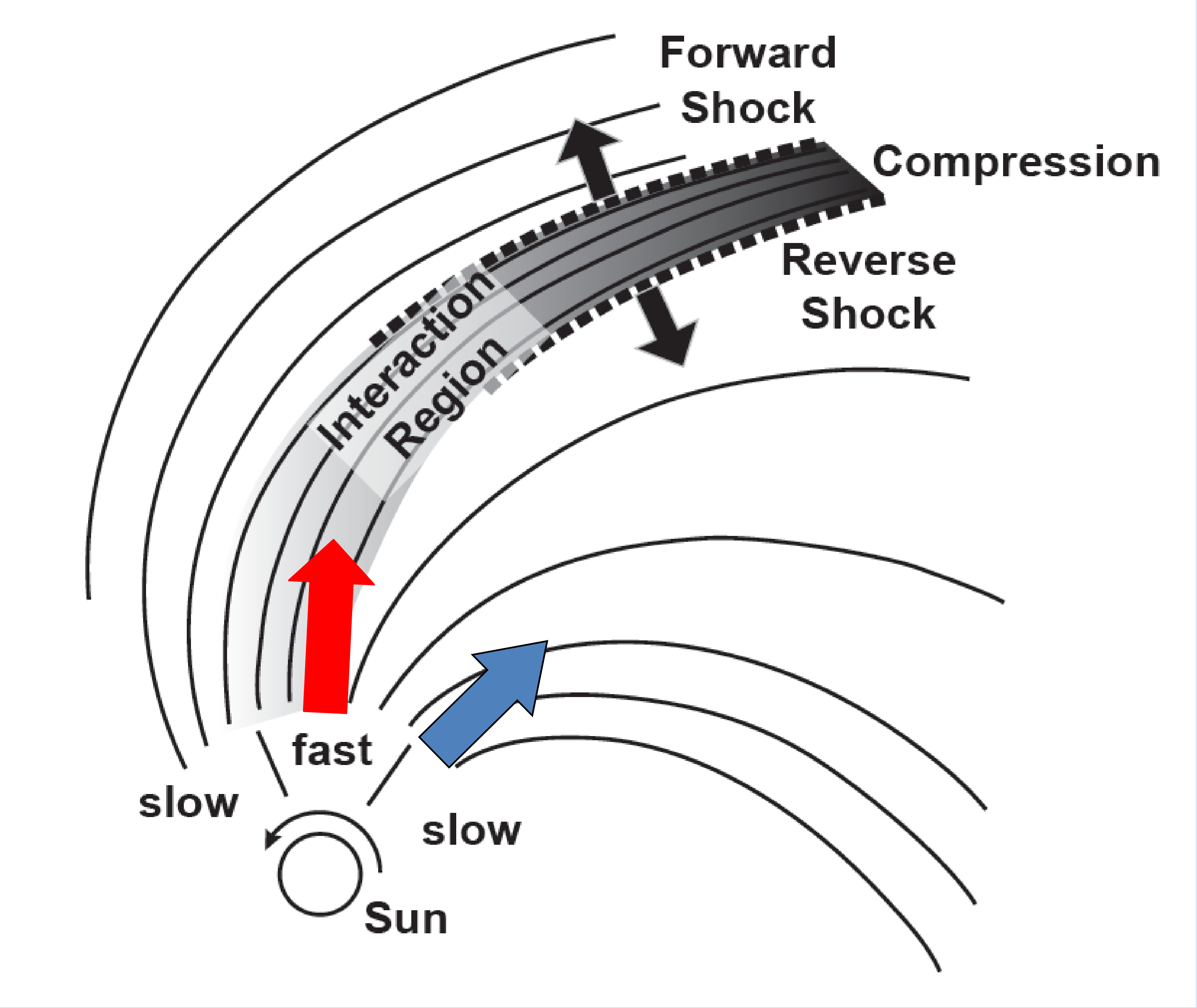
Large CME observed with SOHO coronograph

Solar Energetic Particles (SEP)

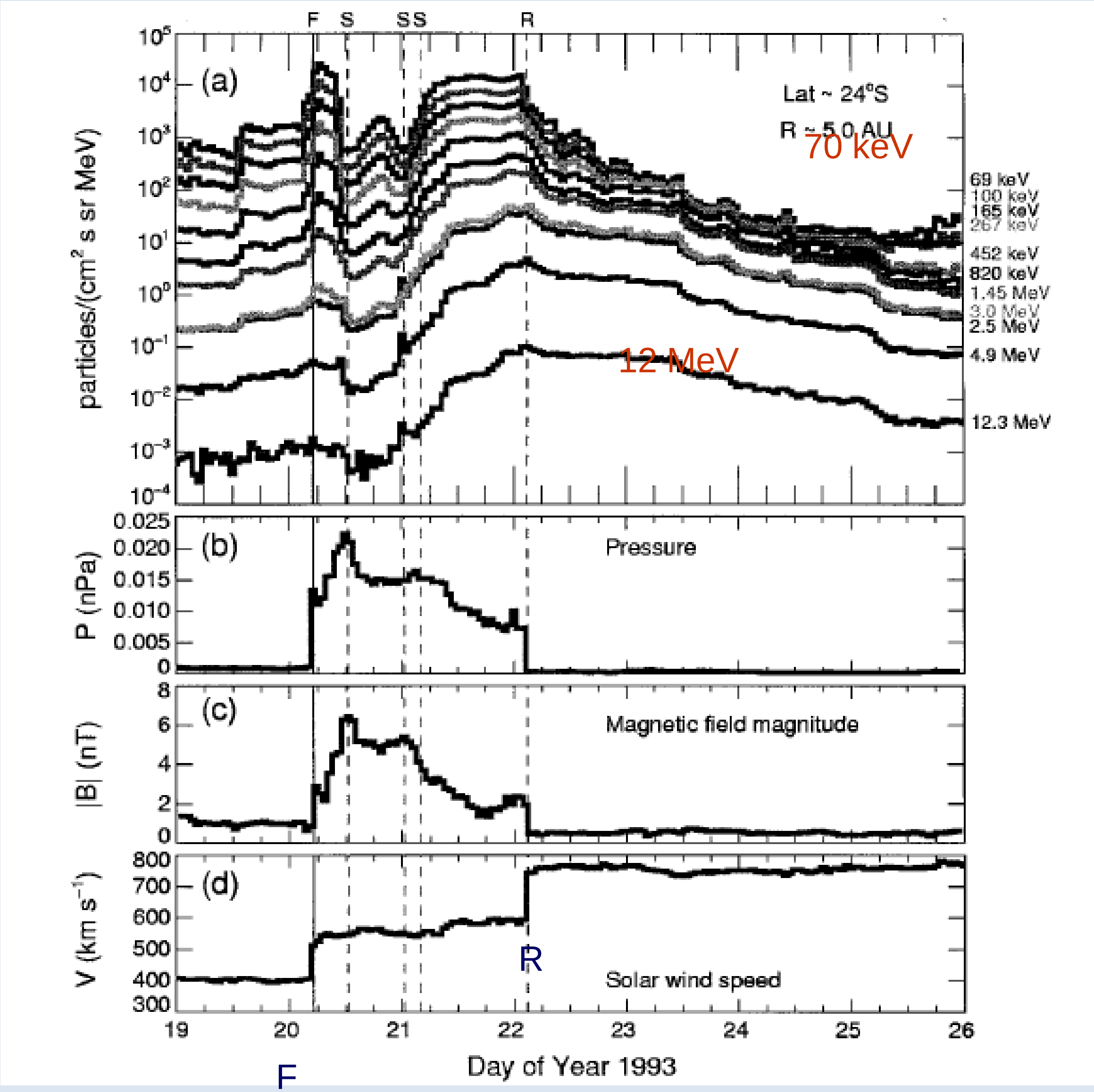


Reames (1999)

Corotating interaction regions and forward and reverse shock

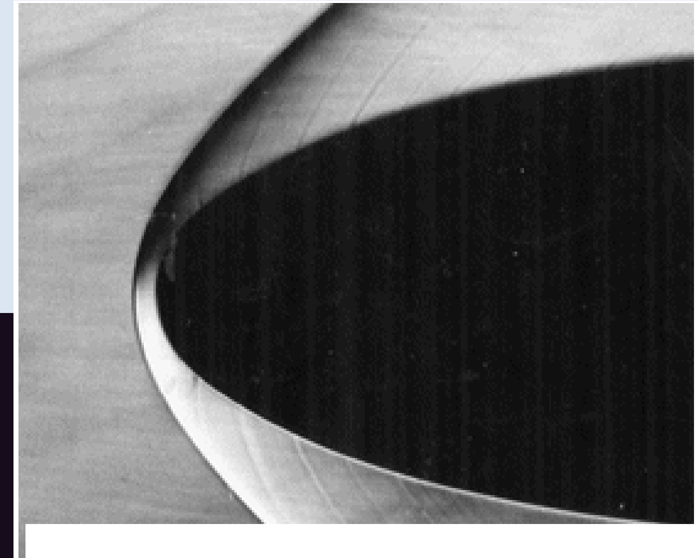


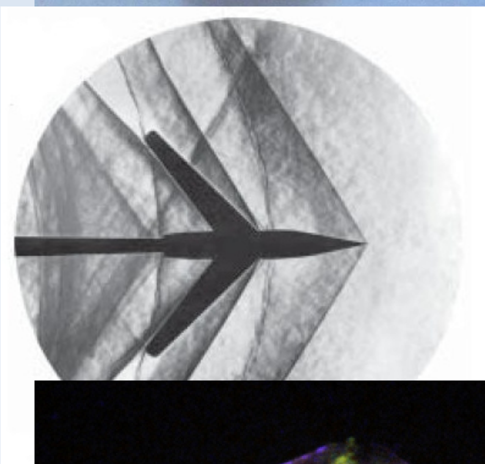
CIR observed by Ulysses at 5 AU

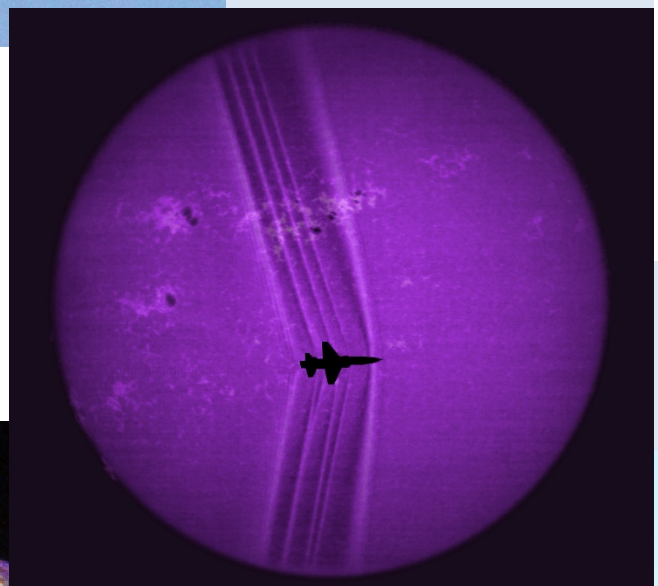


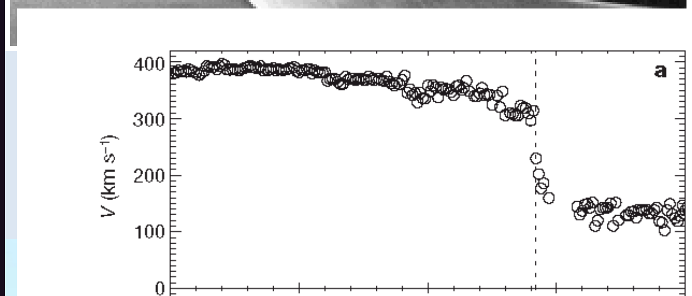
Decker et al. 1999

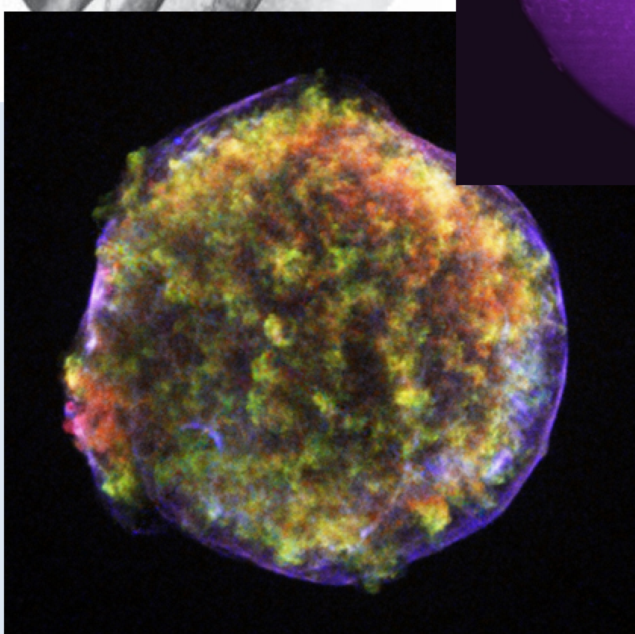
Shocks!



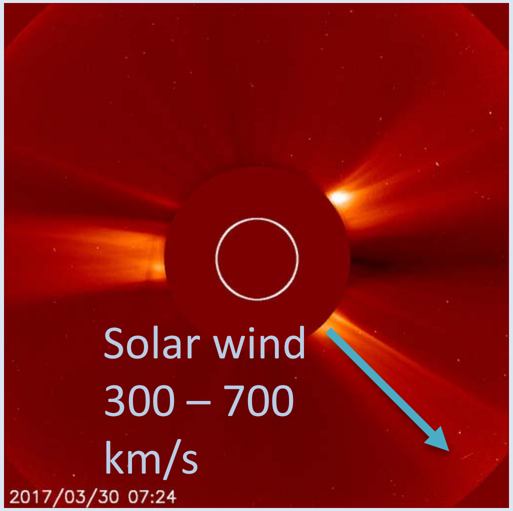








Shocks in Space Plasma



Wave speeds in solar wind ~ 50 km/s

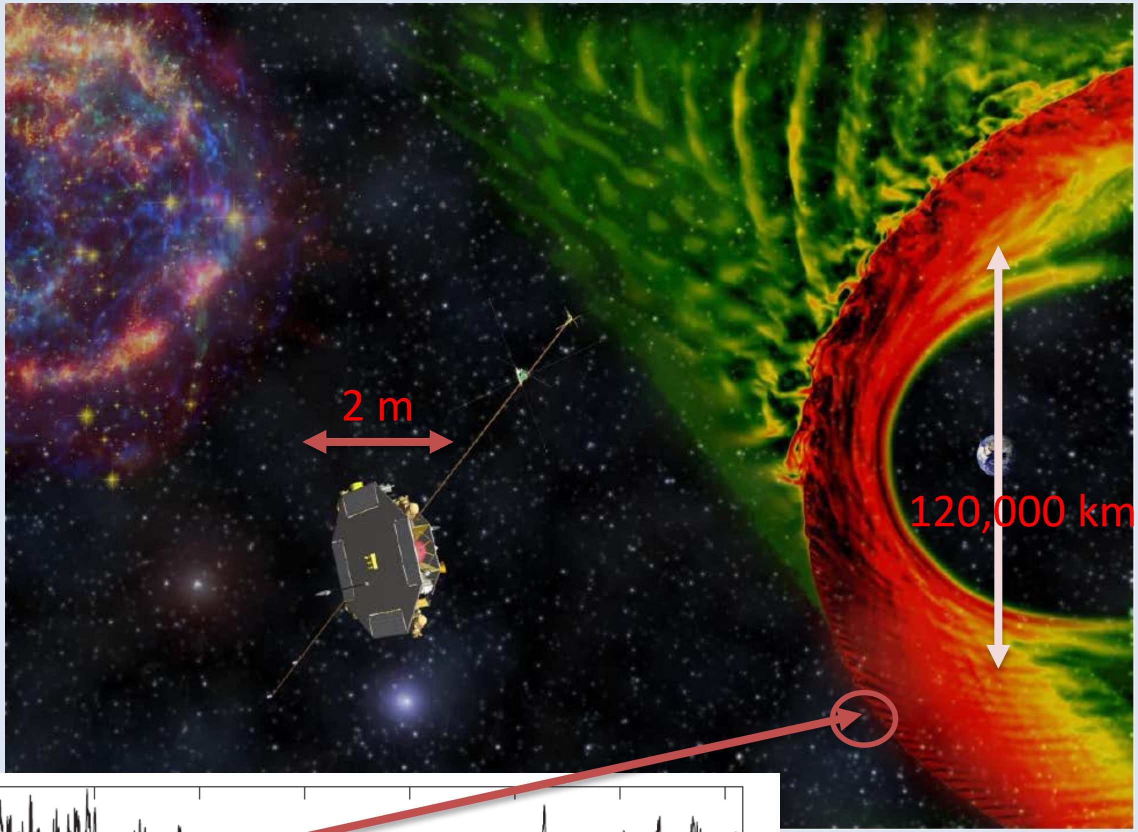
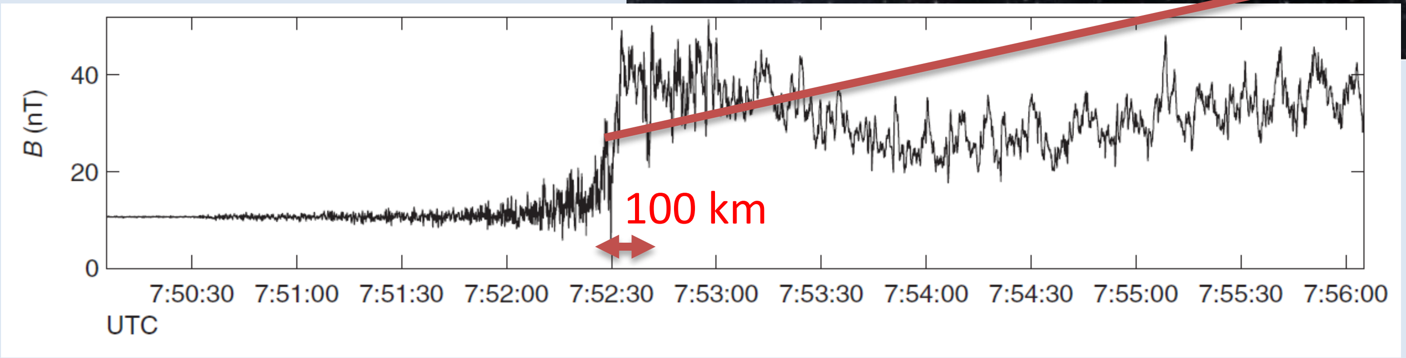


Image: THOR Study Report



Historical Note on Plasma Shocks

PHYSICAL REVIEW VOLUME 80, NUMBER 4 NOVEMBER 15, 1950

Magneto-Hydrodynamic Shocks*

F. DE HOFFMANN AND E. TELLER
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

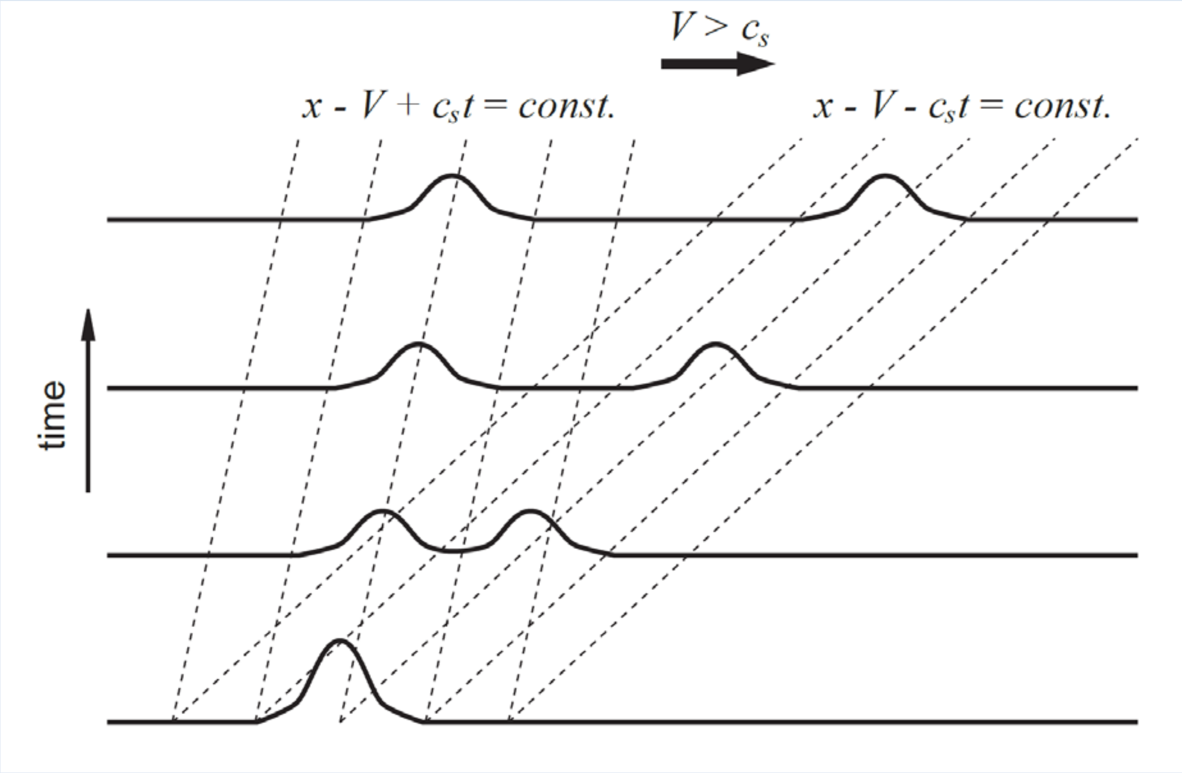
(Received July 10, 1950)

A mathematical treatment of the coupled motion of hydrodynamic flow and electromagnetic fields is given. Two simplifying assumptions are introduced: first, the conductivity of the medium is infinite, and second, the motion is described by a plane shock wave. Various orientations of the plane of the shock and the magnetic field are discussed separately, and the extreme relativistic and unrelativistic behavior is examined. Special consideration is given to the behavior of weak shocks, that is, of sound waves. It is interesting to note that the waves degenerate into common sound waves and into common electromagnetic waves in the extreme cases of very weak and very strong magnetic fields.

Frederic De Hoffmann [& Wife];Edward Teller [& Wife]

Shocks and Nonlinearity

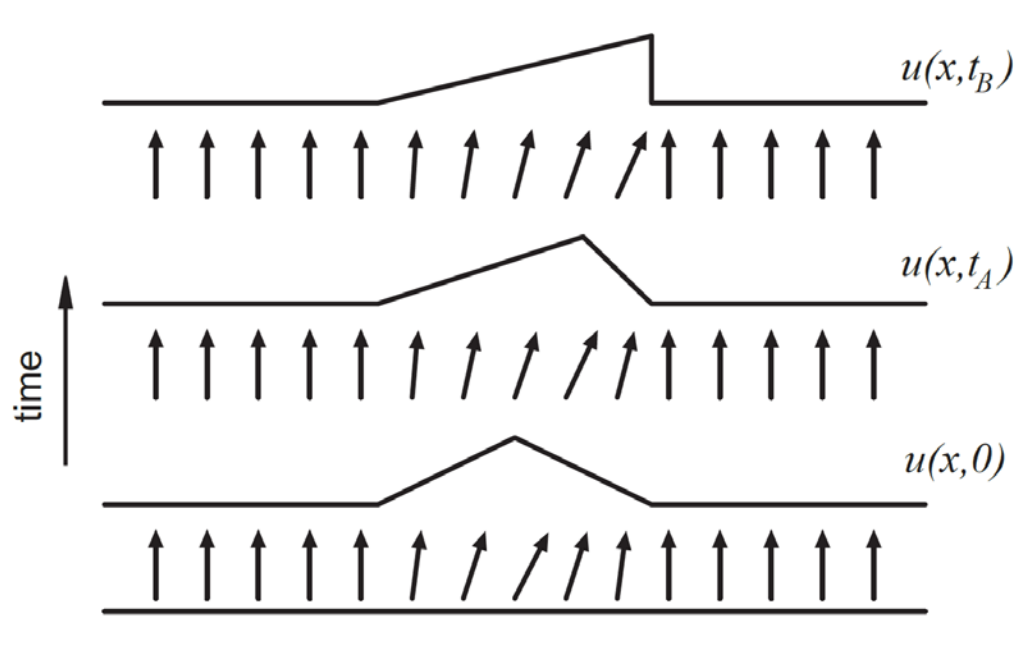
D’Alembert’s solution in a supersonic flow



Burgers' Equation and Steepening

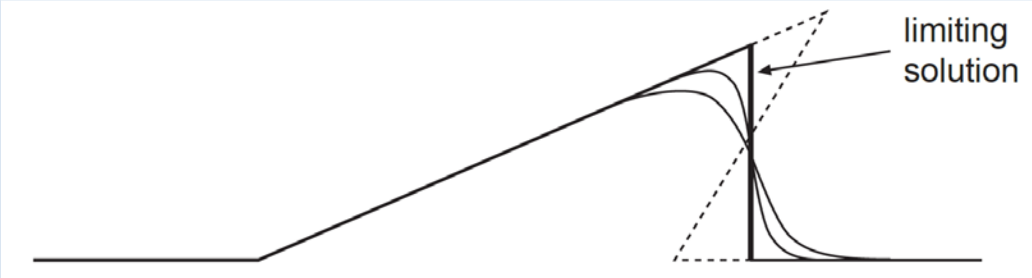
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

Arrows show slope of characteristics

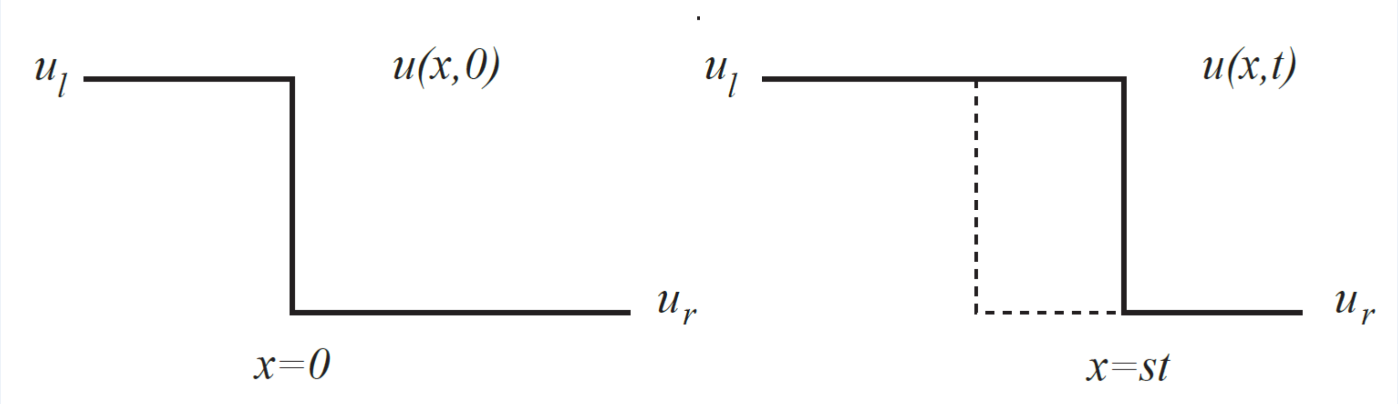


DISSIPATION

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \epsilon\frac{\partial^2 u}{\partial x^2}$$



Burgers' Equation – Discontinuous Solutions



$$u(x,t) = \begin{cases} u_l & x < st \\ u_r & x > st \end{cases}$$

Shock speed s

$$s = \frac{1}{2}(u_l + u_r)$$

Conservation form:

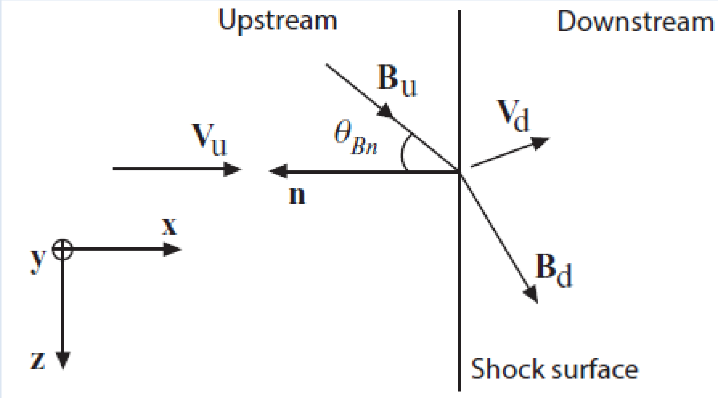
$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{2}u^2\right) = 0$$

Stability requires characteristics sloping *towards* shock

$$u_l > u_r$$

Fluid Shocks

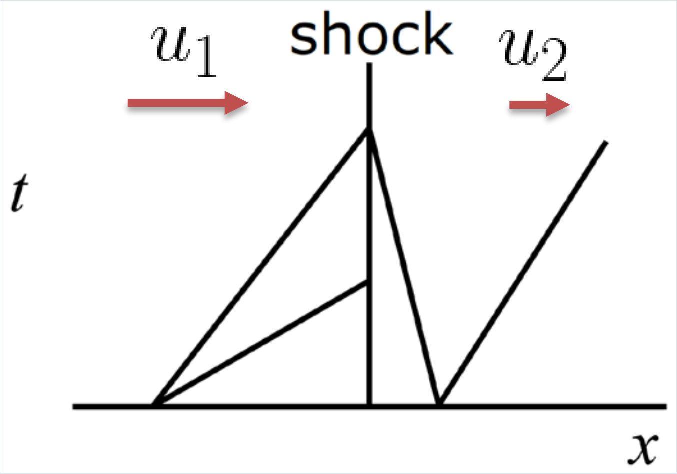
Shock is a (near-) discontinuous solution linking two thermodynamic states



Rankine-Hugoniot

- Unique way of describing downstream state in terms of upstream state and shock parameters (from conservation relations)

To preserve discontinuous solution require characteristics to be directed towards the shock



$$u_1 > c_{su} \text{ and } u_2 < c_{sd}$$

Shock stability - Evolutionary conditions

Is the nature of the discontinuity consistent with the nature of the boundary conditions (described by upstream and downstream characteristics)?

For simple fluid – the evolutionary shock with supersonic to subsonic transition has increase of entropy

Conservation Relations for Vlasov System

Moments

$$n = \int \langle f \rangle \, d\mathbf{v},$$

$$\mathbf{V} = \int \mathbf{v} \langle f \rangle \, d\mathbf{v},$$

$$\mathsf{P} = m \int \langle f \rangle (\mathbf{v} - \mathbf{V})(\mathbf{v} - \mathbf{V}) \, d\mathbf{v},$$

Assuming a suitable closure leads to MHD (and MHD conservation relations)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) (p \rho^{-\gamma}) = 0$$

For Vlasov, with time stationarity, a similar set of equations

$$\int v_x \langle f_j \rangle \, d\mathbf{v} = \text{const.},$$

$$\sum_j \int m_j v_x \, \mathbf{v} \, \langle f_j \rangle \, d\mathbf{v} + \frac{\epsilon_0 \widehat{\mathbf{x}}}{2} (\langle \mathbf{E} \rangle^2 + c^2 \langle \mathbf{B} \rangle^2 + \langle \delta \mathbf{E}^2 \rangle + c^2 \langle \delta \mathbf{B}^2 \rangle)$$
$$- \epsilon_0 (\langle \mathbf{E} \rangle \langle E_x \rangle + c^2 \langle \mathbf{B} \rangle \langle B_x \rangle + \langle \mathbf{E} \, \delta E_x \rangle + c^2 \langle \mathbf{B} \, \delta B_x \rangle) = \text{const.},$$

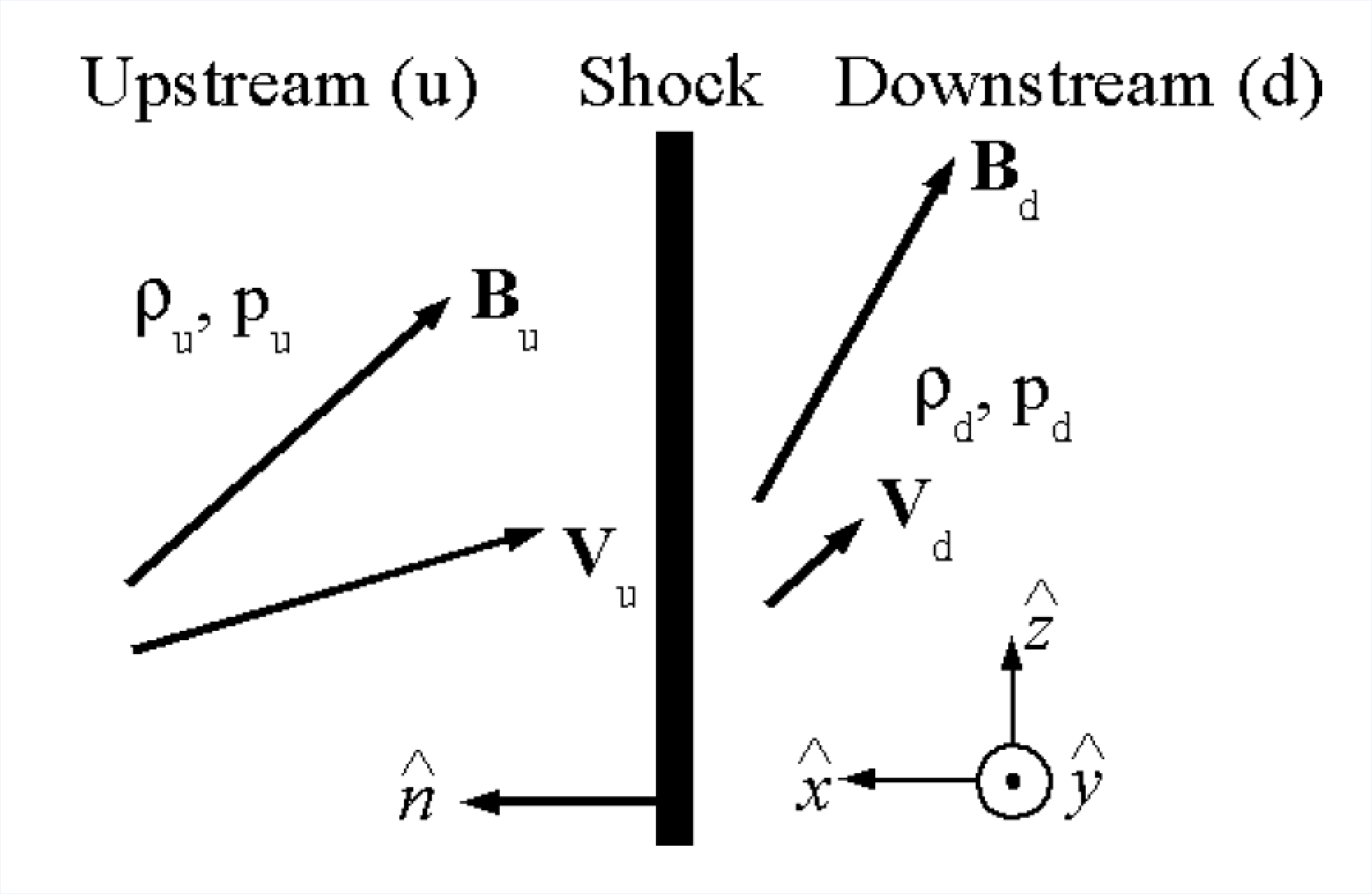
$$\sum_j \int \frac{1}{2} m_j v^2 \, v_x \, \langle f_j \rangle \, d\mathbf{v} + \frac{1}{\mu_0} \widehat{\mathbf{x}} \cdot (\langle \mathbf{E} \rangle \times \langle \mathbf{B} \rangle + \langle \delta \mathbf{E} \times \delta \mathbf{B} \rangle) = \text{const.}$$

$$\int v_x \, \langle \phi(f) \rangle \, d\mathbf{v} = \text{const.}$$

Need to have form of <f>

- Maxwellians!
- Equal ion and electron temperature
- And then recover MHD conservation relations!

MHD CONSERVATION RELATIONS



MHD CONSERVATION RELATIONS

- MHD conservation relations lead to complicated set of different types of discontinuity

- Evolutionary conditions complicated by multiple wave modes and fact that two sets of conditions have to be solved (Alfvenic perturbations decouple from magnetosonic and entropy)

- For shocks (with mass flux across surface) only FAST and SLOW mode shocks are evolutionary

  FAST: $V_{n,u} > V_{f,u}$ and $V_{f,d} > V_{n,d} > V_{A,d}$

  SLOW: $V_{A,u} > V_{n,u} > V_{s,u}$ and $V_{s,d} > V_{n,d}$

- Strong shocks are almost definitely fast mode shocks!

$$[\rho V_x] = 0$$

$$\left[\rho V_x^2 + p + \frac{B^2}{2\mu_0}\right] = 0$$

$$\left[\rho V_x \mathbf{V}_t - \frac{B_x}{\mu_0}\mathbf{B}_t\right] = 0$$

$$\left[\rho V_x \left(\frac{1}{2}V^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}\right) + V_x\frac{B^2}{\mu_0} - \mathbf{V}\cdot\mathbf{B}\frac{B_n}{\mu_0}\right] = 0$$

$$[B_x] = 0$$

$$[V_x\mathbf{B}_t - B_x\mathbf{V}_t] = 0$$

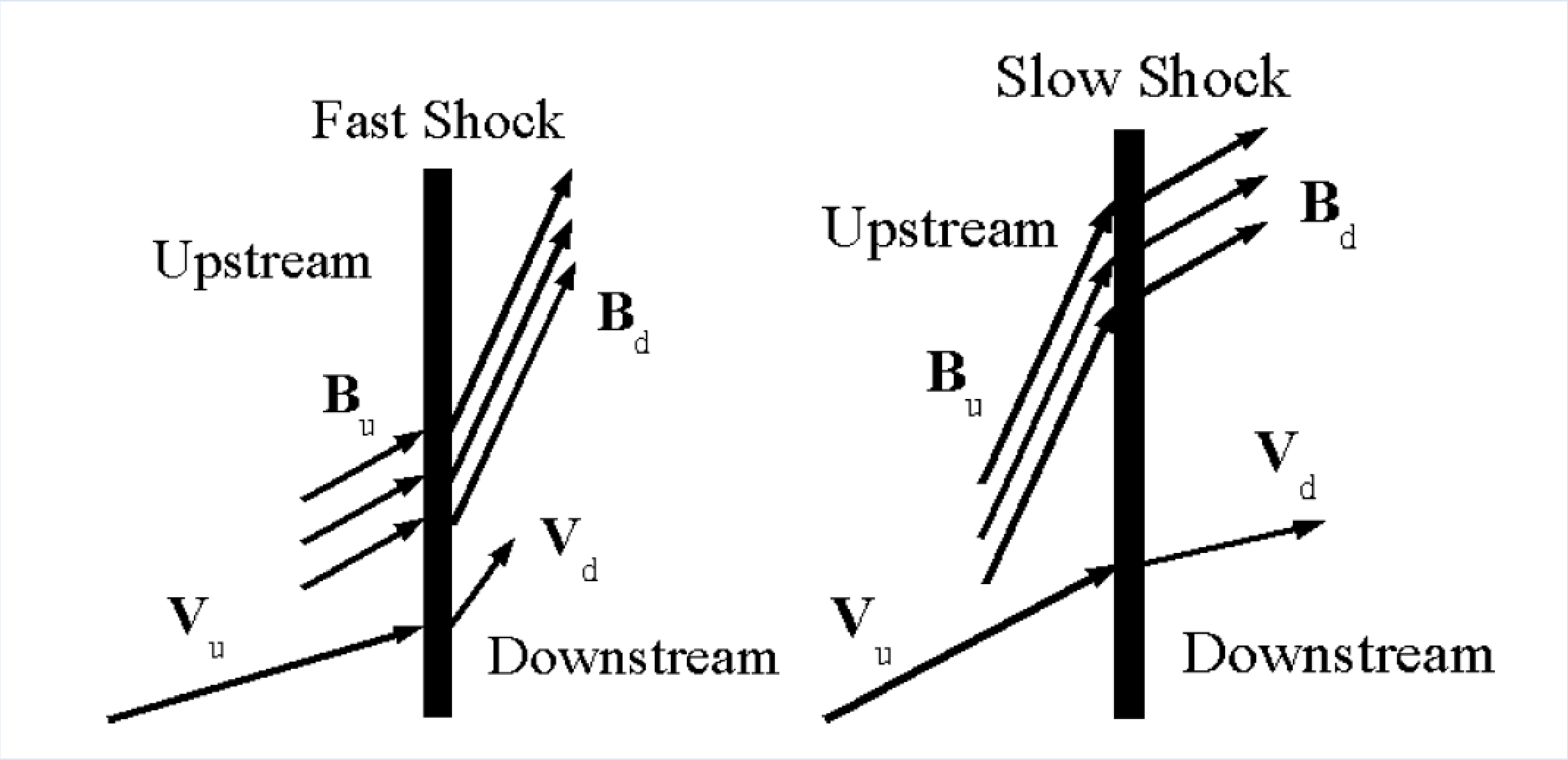
Discontinuities

Contact Discontinuity	$\mathbf{V}_u = \mathbf{0}$, $B_n \neq 0$	Density jump arbitrary, but pressure and all other quantities are continuous.
Tangential Discontinuity	$V_x = 0$, $B_n = 0$	Plasma pressure and field change maintaining static pressure balance.
Rotational Discontinuity	$V_n = B_n/\sqrt{\mu_0 \rho}$	Form of intermediate shock in isotropic plasma, field and flow change direction but not magnitude.

Shock Waves: $\mathbf{V}_u \neq \mathbf{0}$

Parallel Shock	$B_t = 0$	Magnetic field unchanged by shock.
Perpendicular Shock	$B_n = 0$	Plasma pressure and field strength increases at shock.
Oblique Shocks	$B_t \neq 0$, $B_n \neq 0$	
Fast Shock		Plasma pressure and field strength increase at shock, magnetic field bends away from normal.
Slow Shock		Plasma pressure increases, magnetic field strength decreases, magnetic field bends towards normal.
Intermediate Shock		Only shocklike in anisotropic plasma.

The collisionless caveat: Any additional parameter (temperature anisotropy, heat flux, unequal temperatures, wave energy) makes the shock conservation relations incomplete – and some additional assumptions have to be made!



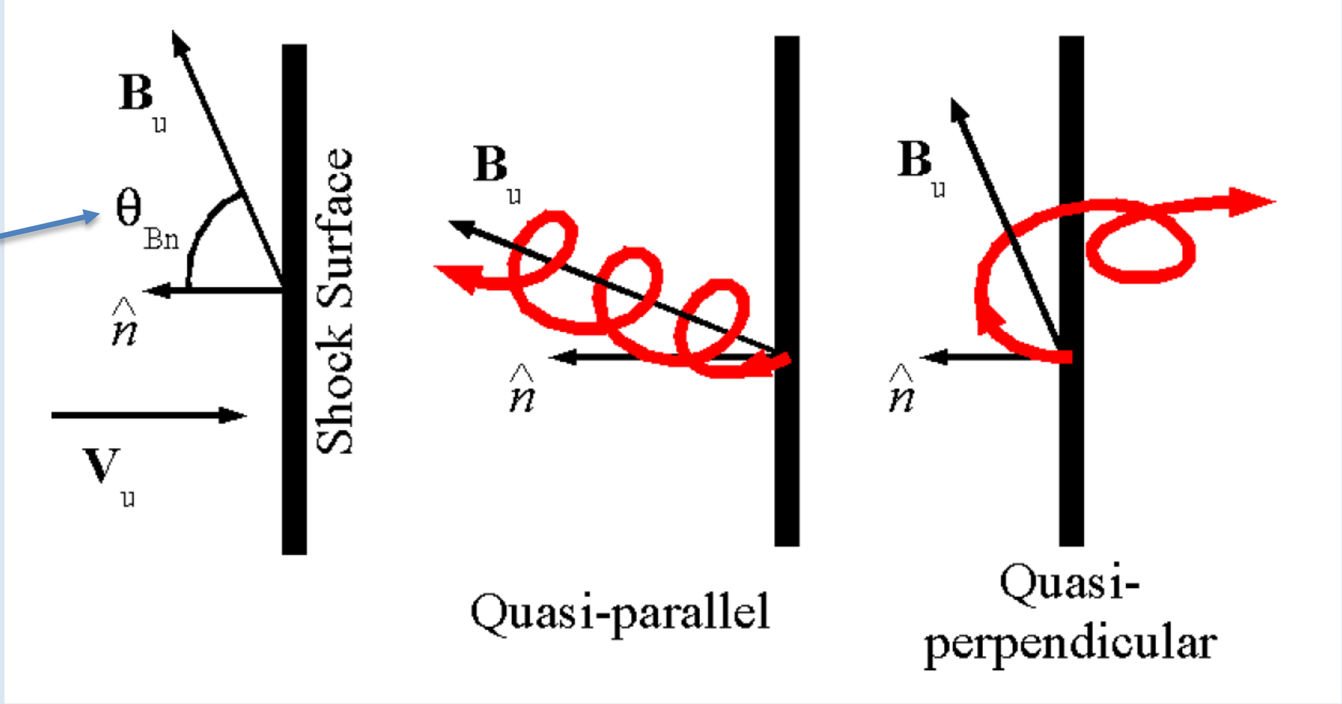
$V_{n,u} > V_{f,u}$ and $V_{f,d} > V_{n,d} > V_{A,d}$

$V_{A,u} > V_{n,u} > V_{s,u}$ and $V_{s,d} > V_{n,d}$

SHOCK PARAMETERS

Shock normal angle

θ_{Bn}



Alfvén Mach Number $$M_A = \frac{V_u}{B_u/\sqrt{\mu_0 \rho_u}}$$

Other plasma parameters: plasma beta, composition, anisotropy, etc!

USING THE SHOCK CONSERVATION RELATIONS

Exactly parallel shock

$$\mathbf{B}_u = B_x\hat{n},\ \mathbf{B}_{ut} = \mathbf{0}$$

Using transverse momentum equation

$$\left[\left(1 - \frac{B_n^2}{\mu_0 \rho V_n^2}\right) V_n \mathbf{B}_t\right] = \mathbf{0}$$

Transverse magnetic field zero upstream and downstream, so magnetic field is unchanged by shock.

Shock jump relations same as for an ordinary gas shock.

USING THE SHOCK CONSERVATION RELATIONS

Exactly perpendicular shock

$$B_x = 0 \text{ and } \mathbf{B}_u = \mathbf{B}_{ut}$$

From conservation of tangential electric field

$$V_{ux}\mathbf{B}_{ut} = V_{dx}\mathbf{B}_{dt}$$

density *compression ratio* $r = \rho_d / \rho_u$

Magnetic field compresses as much as the density

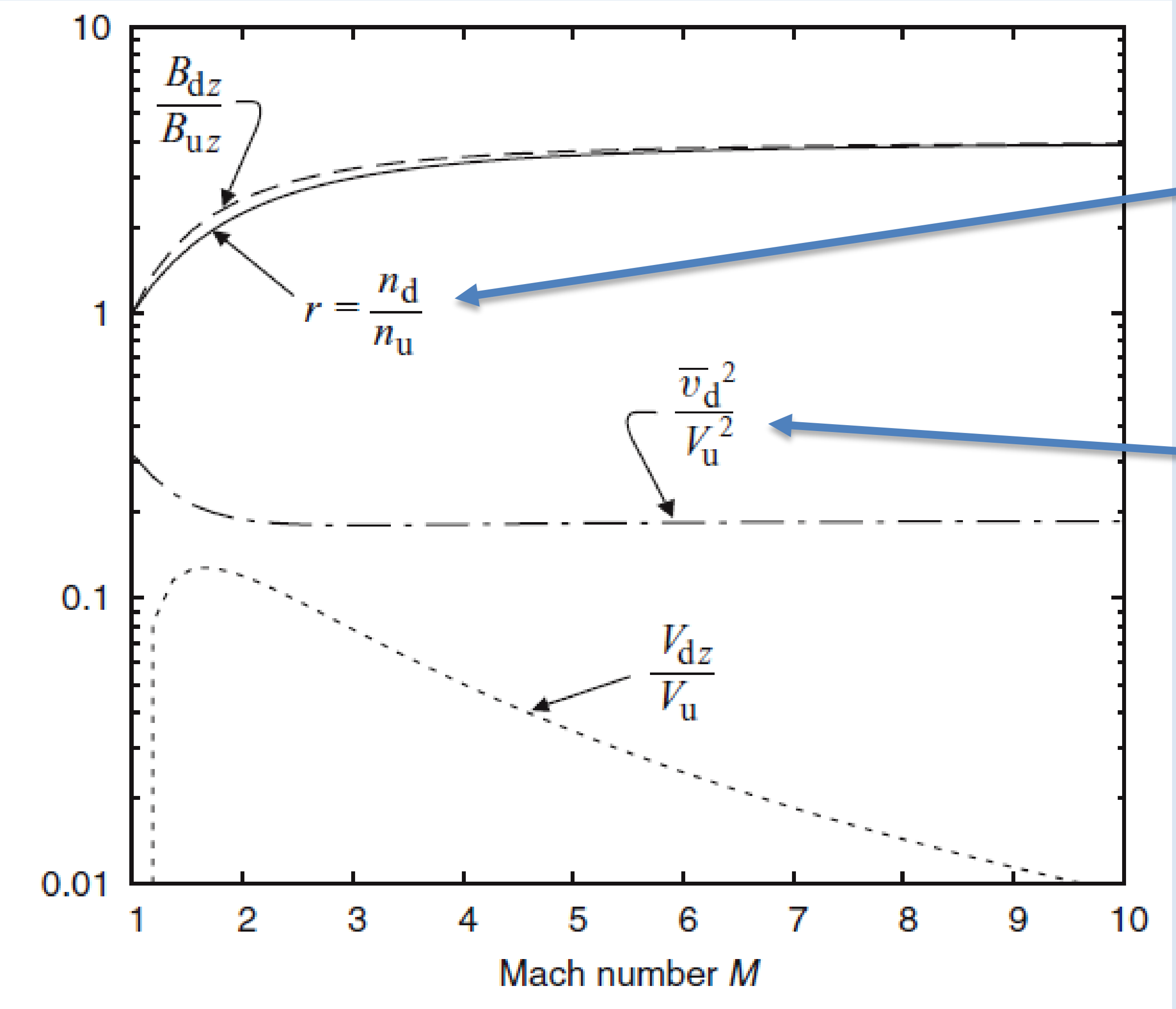
For high Mach number, compression ratio becomes

$$r = \frac{\gamma + 1}{\gamma - 1}$$

$M_A \gg 1$ and $M_{cs} \gg 1$

Compression ratio has limit of about 4 as Mach number increases

USING THE SHOCK CONSERVATION RELATIONS



Compression ratio

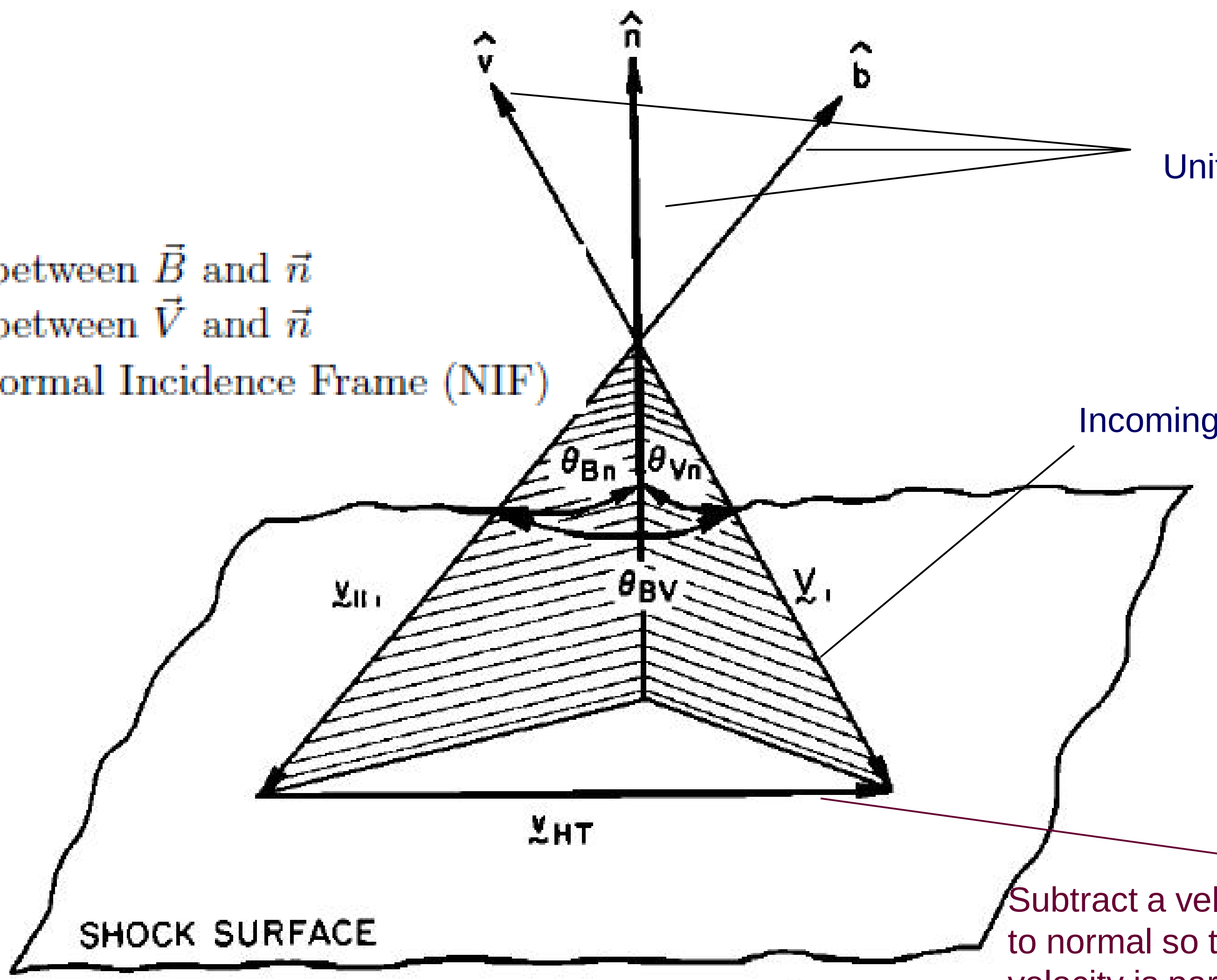
Ratio downstream thermal speed to upstream flow speed

de Hoffmann-Teller Frame (HTF) and Normal Incidence Frame (NIF)

θ_{Bn} = angle between $\vec{B}$ and $\vec{n}$

θ_{Vn} = angle between $\vec{V}$ and $\vec{n}$

If $\theta_{Vn} = 0$: Normal Incidence Frame (NIF)



The de Hoffmann-Teller velocity

$\vec{V}_{in}$ = incoming velocity, $\vec{V}_{HT}$ = de Hoffmann-Teller velocity, $\vec{V}_{in}^*$ = incoming velocity in the H-T frame

$$\vec{V}_{in} = \vec{V}_{HT} + \vec{V}_{in}^*$$

Take cross-product with $\vec{B}$

$$\vec{B} \times \vec{V}_{in}^* = 0 = \vec{B} \times \vec{V}_{in} - \vec{B} \times \vec{V}_{HT}$$

take cross-product with $\vec{n}$

$$\vec{n} \times (\vec{B} \times \vec{V}_{HT}) = \vec{n} \times (\vec{B} \times \vec{V}_{in})$$

$$\vec{n} \times (\vec{B} \times \vec{V}_{HT}) = \vec{B}(\vec{n} \cdot \vec{V}_{HT}) - \vec{V}_{HT}(\vec{n} \cdot \vec{B})$$

$$\vec{V}_{HT} = -\frac{\vec{n} \times (\vec{B} \times \vec{V}_{in})}{\vec{n} \cdot \vec{B}}$$

de HOFFMAN-TELLER FRAME

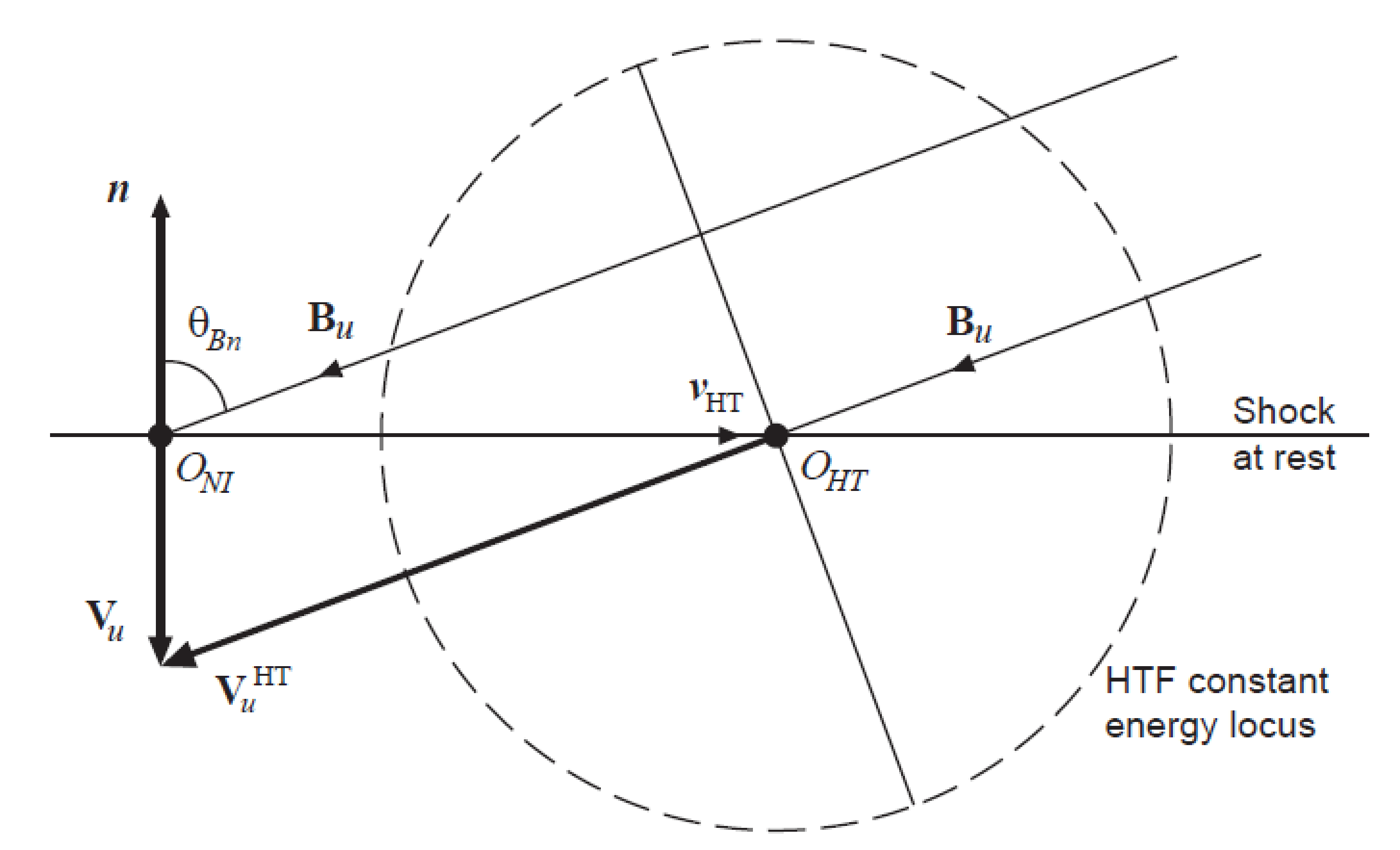
1. Since in the de Hoffmann -Teller (HT) frame the velocity is parallel to the magnetic field the $\vec{V} \times \vec{B}$ electric field vanishes in that frame

2. In the HT frame the shock is at rest - no induced electric field

A consequence of $\vec{E} = 0$ is that the energy of a particle is constant, and surfaces of constant particle energy are spheres centered on the HT frame origin

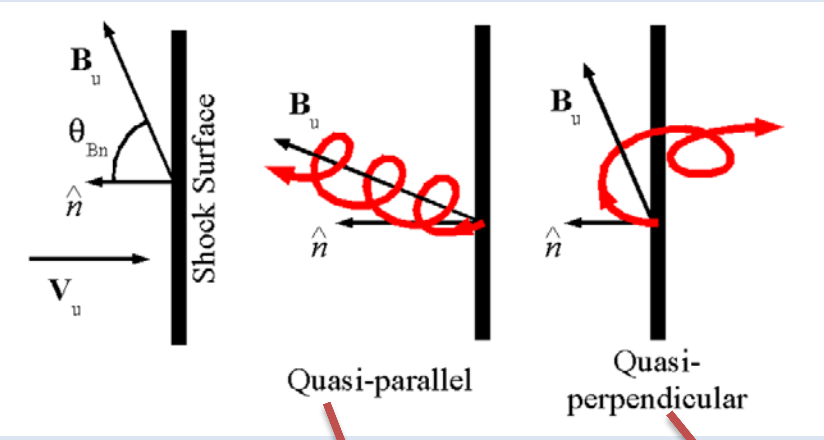
3. The de Hoffmann-Teller transformation velocity is the same downstream as it is upstream

de HOFFMAN-TELLER FRAME



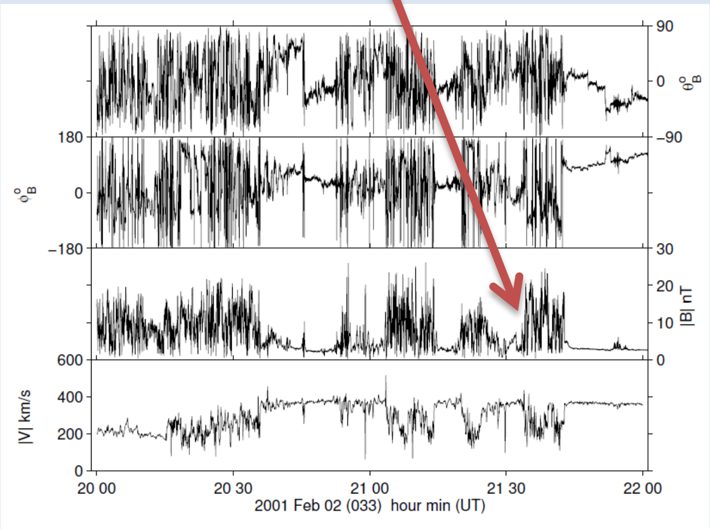
$$v_{\mathrm{HT}} = V_u \tan\theta_{Bn}$$

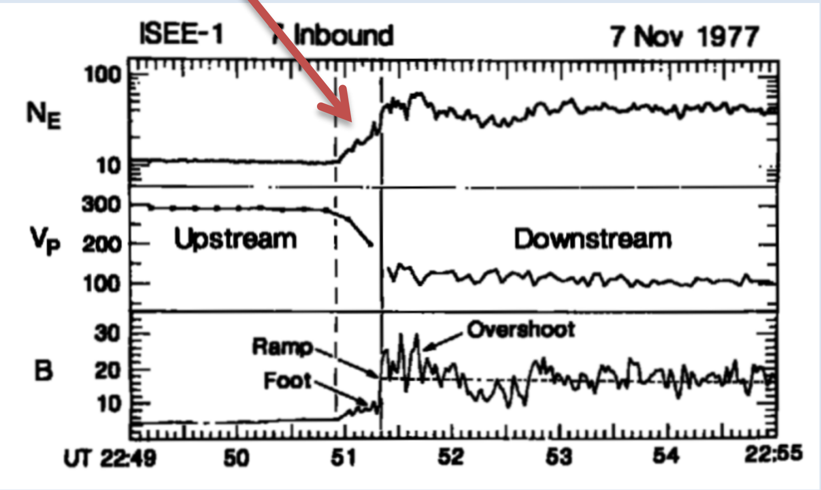
Collisionless Shock Structure – Shock Scale Lengths



Processes:

- Anomalous resistivity (etc)
- Wave Dispersion
- Particle kinetics
- Foreshock-shock interactions



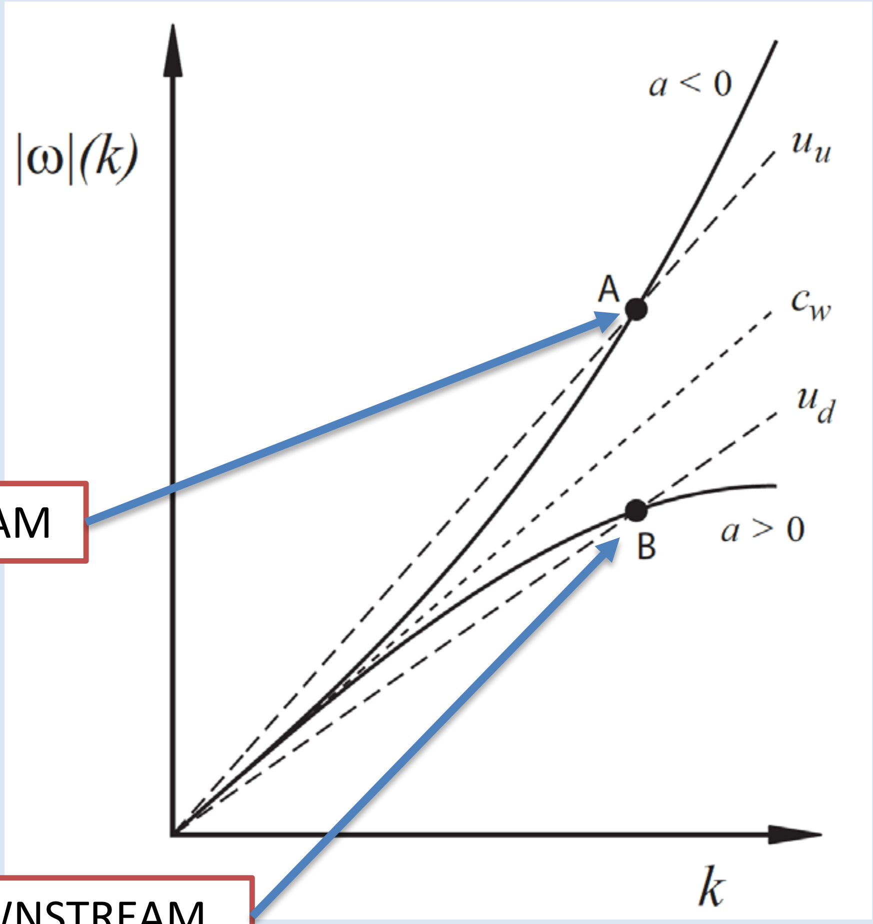


KdV Equation and Dispersion

$$\frac{\partial u}{\partial t} + (u \pm c_w)\frac{\partial u}{\partial t} = a\frac{\partial^3 u}{\partial x^3}$$

Dispersion relation:

$$\omega = \pm k\, c_w + a k^3$$

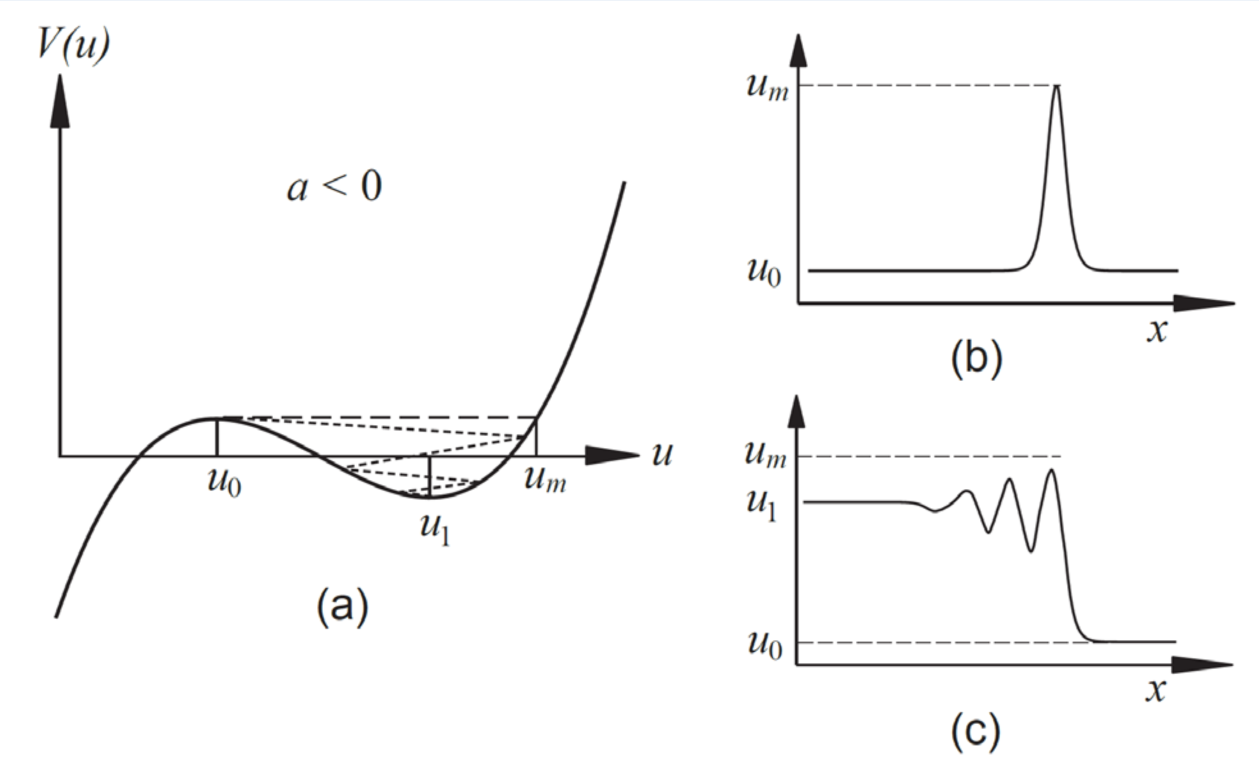


Phase standing UPSTREAM

Phase standing DOWNSTREAM

For shock, need wave speed less than upstream flow and greater than downstream flow

KdV Equation: Solitons and Wavetrains



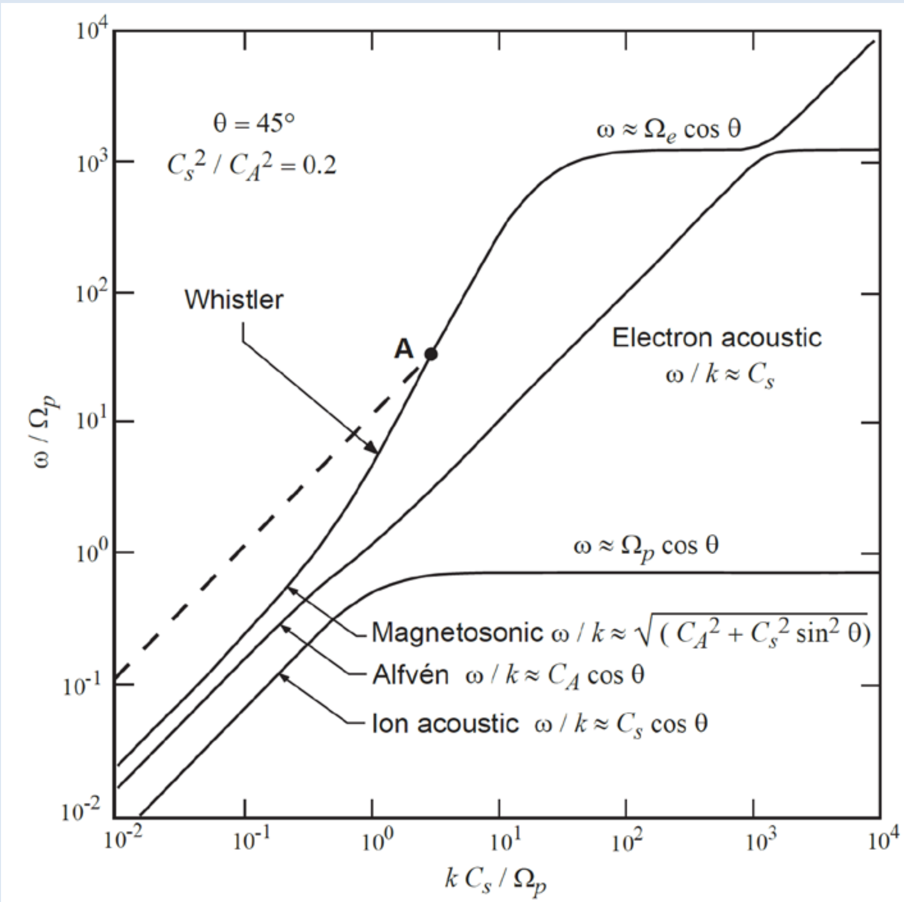
Solution via nonlinear oscillator equation:

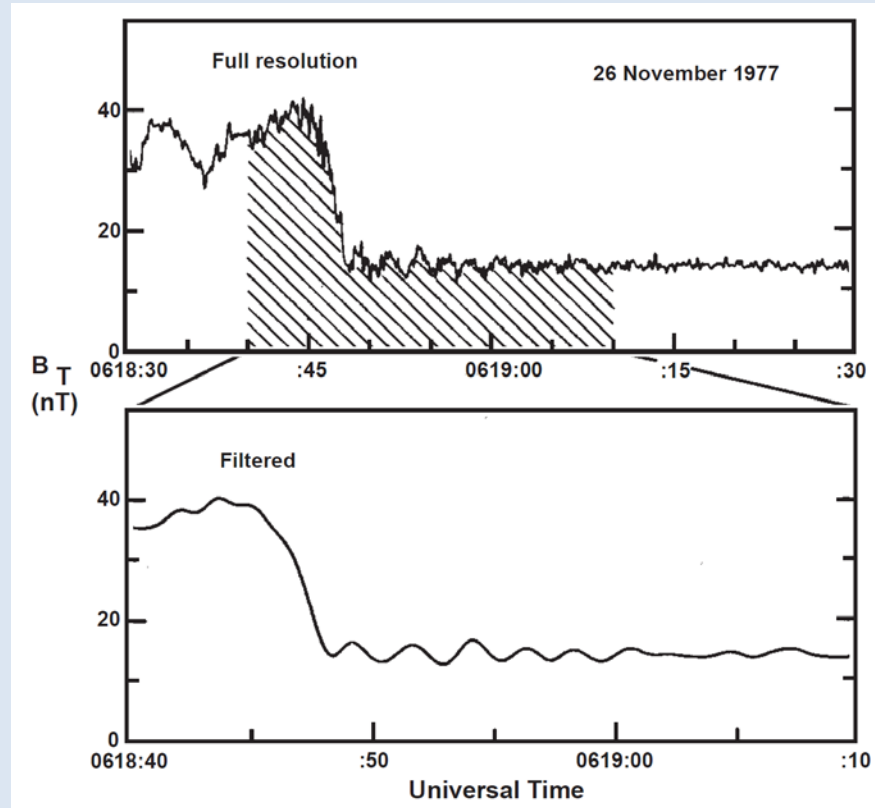
$$\frac{d^2u}{dx^2} = -\frac{dV(u)}{du},$$

$$V(u) = -\frac{1}{6a}\,(u - c_w)^3 + b\,(u - c_w)$$

KdV equation has SOLITON and WAVETRAIN solutions
Dissipation allows shock-like solutions with trailing/leading damped wavetrains

Phase Standing Whistler

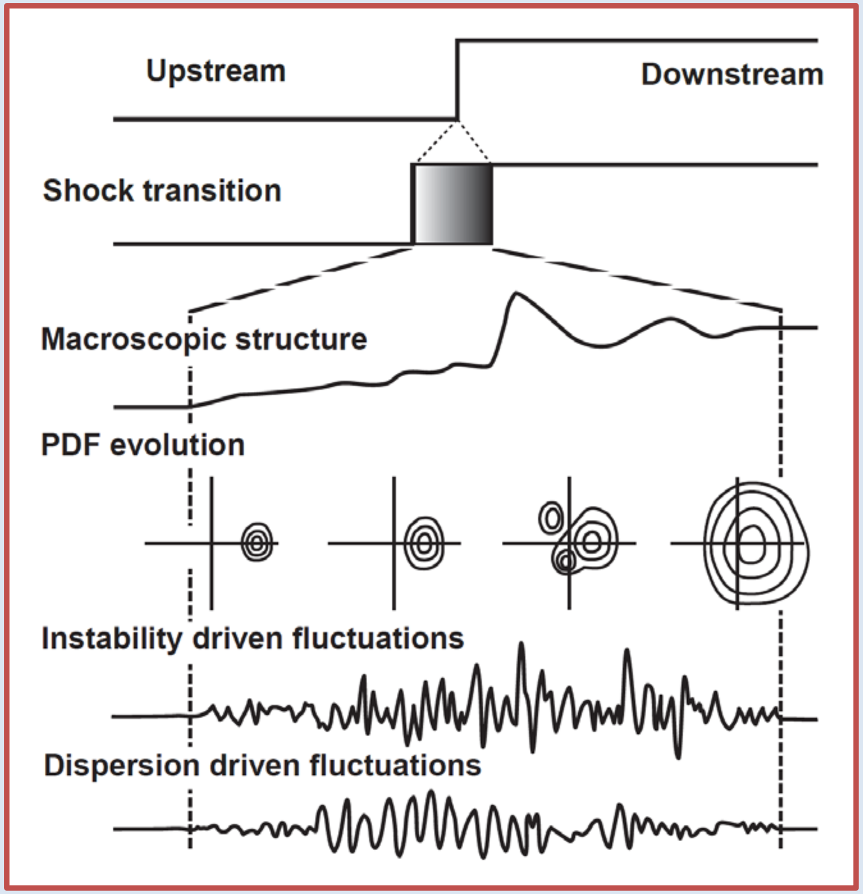




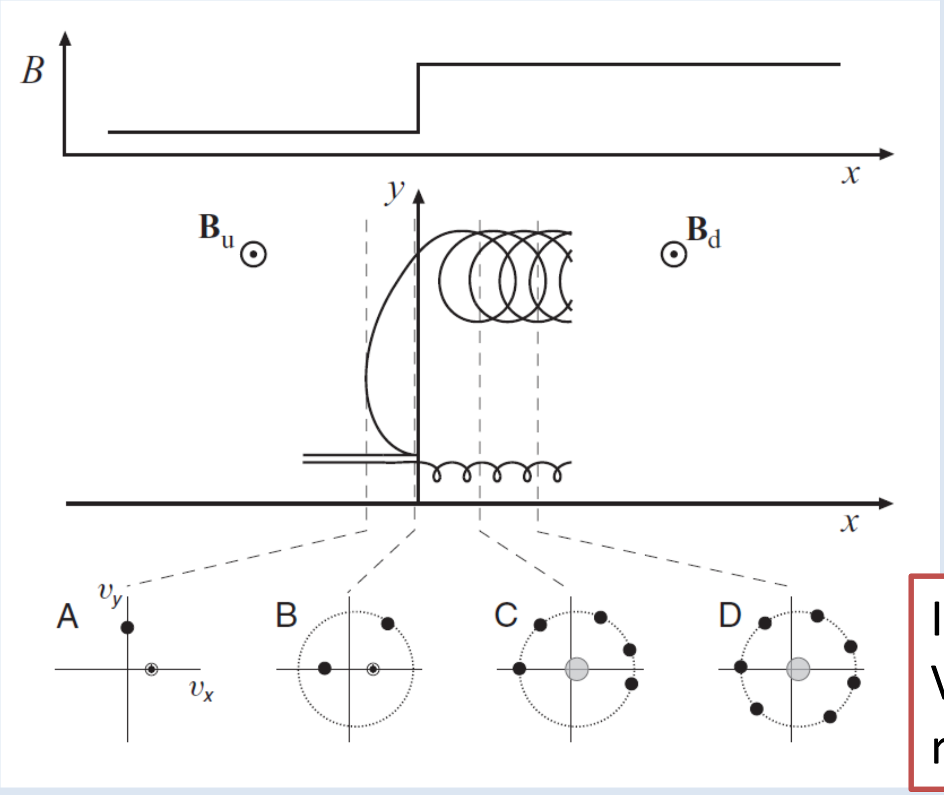
Low Mach number oblique bow shock

Collisionless Shock – How?

Consider processes for
- Velocity space spreading
- Heating/dissipation
- Return to stability



From macro- to microscopic



ION REFLECTION:
Velocity space spreading by reflection and gyration

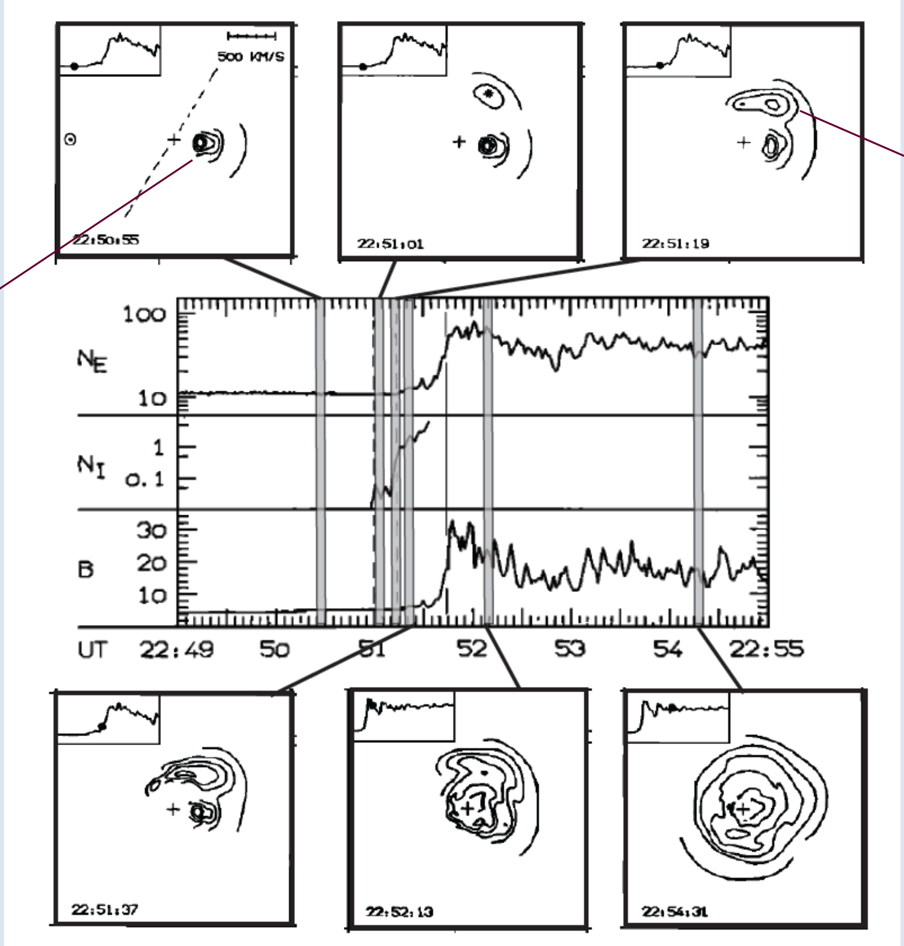
ION REFLECTION AND SHOCK STRUCTURE

QUASI-PERPENDICULAR

Ion reflection leads to a heated downstream distribution

Sun ←

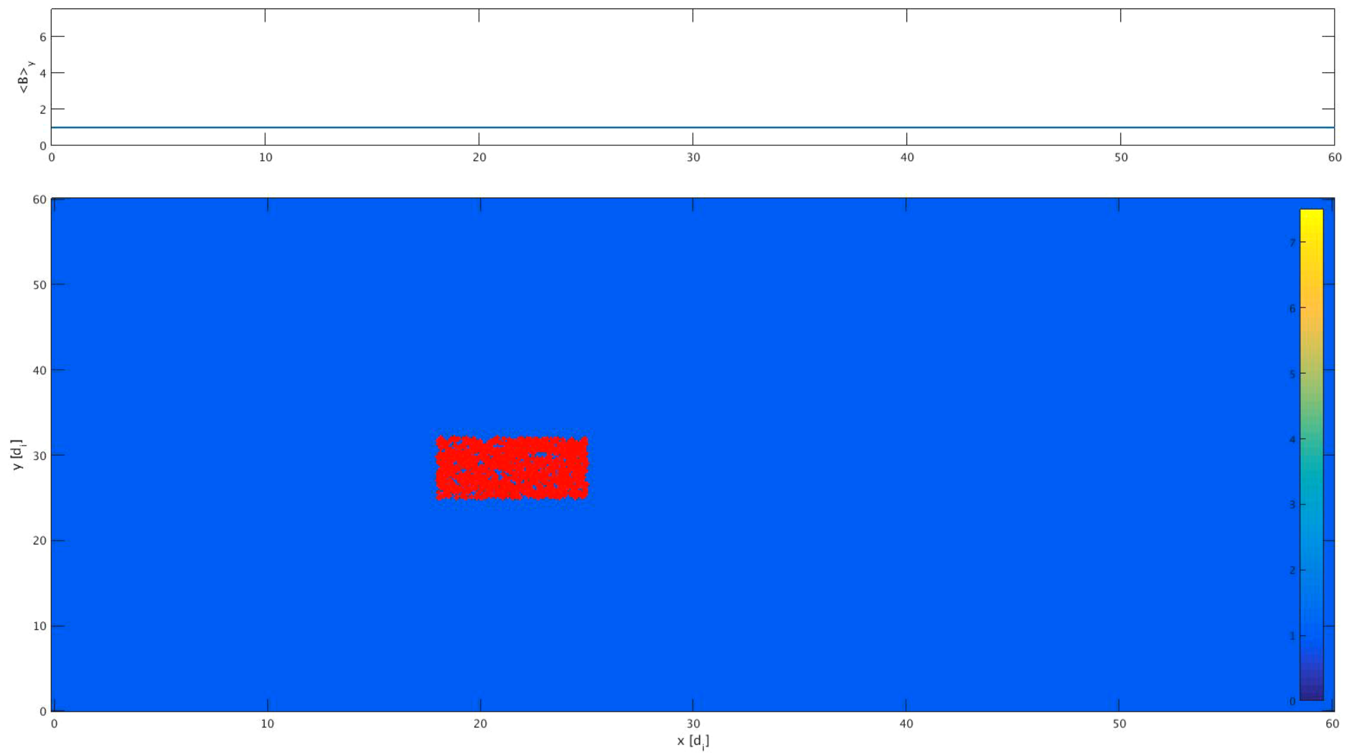
Solar wind



Specularly reflected ions

Adapted from Sckopke et al 1983

ION REFLECTION AND SHOCK STRUCTURE



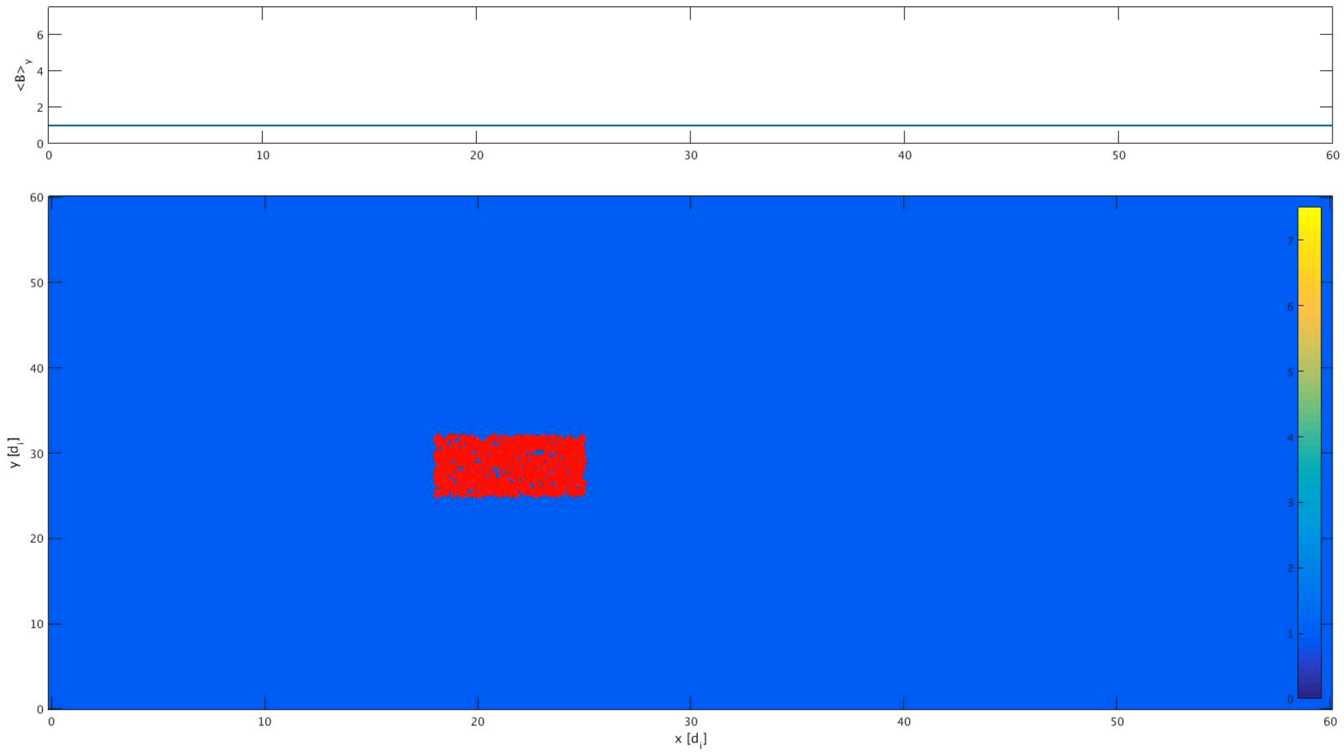
PIC Hybrid (2D) simulation

- High Mach number
- Particles shown are ions
- B field is OUT OF simulation plane

Movie: D. Trotta

- Reflection and gyration produce “heating”
- Reflected ions are shifted in position
- Spatial mixing depends on velocity space – very “Vlasov”!

ION REFLECTION AND SHOCK STRUCTURE



PIC (2D) simulation of Vlasov plasma

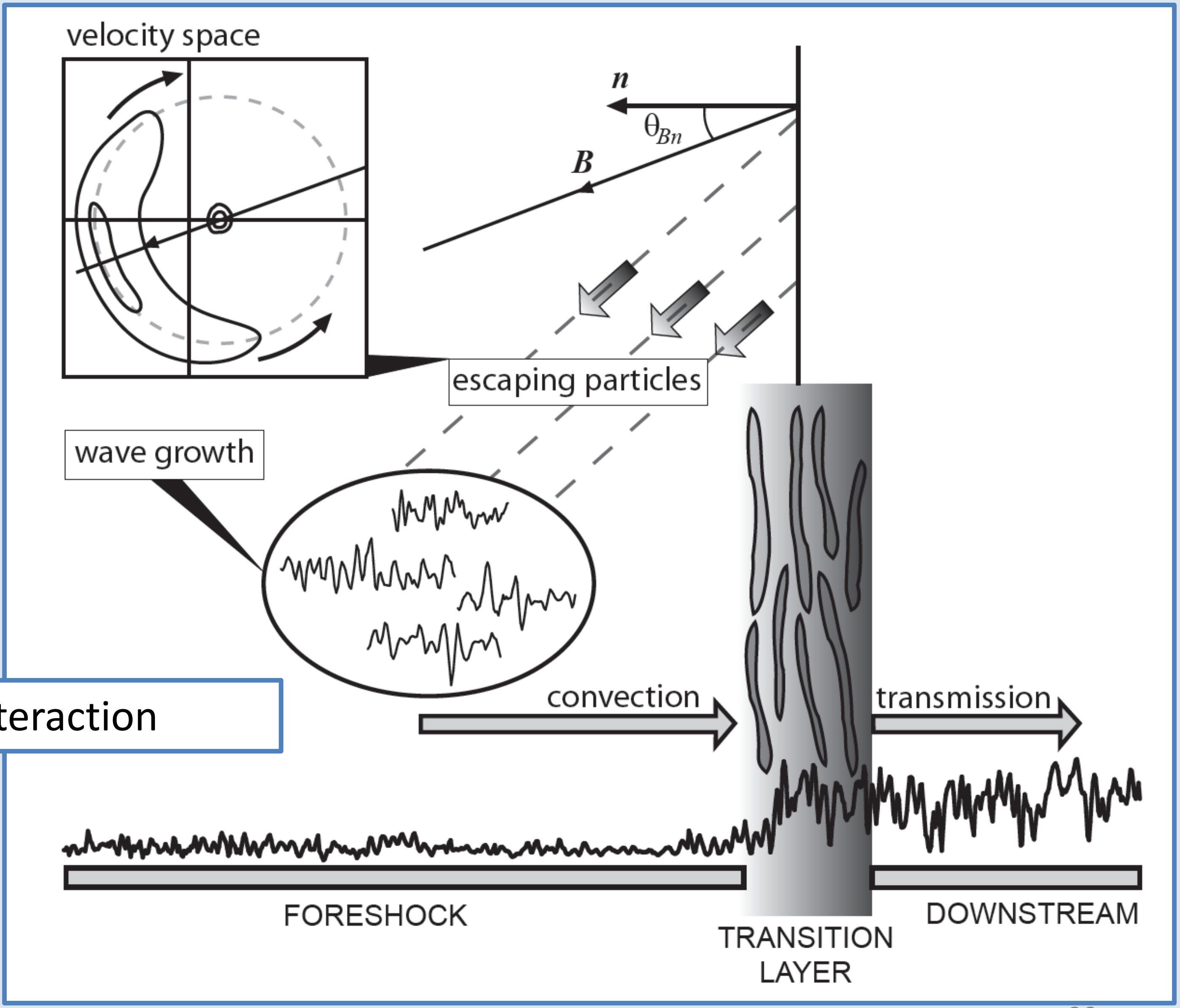
- B field is IN simulation plane

- Spreading of ions in magnetic field direction

- Scattered in downstream waves

- Isotropization and spatial mixing

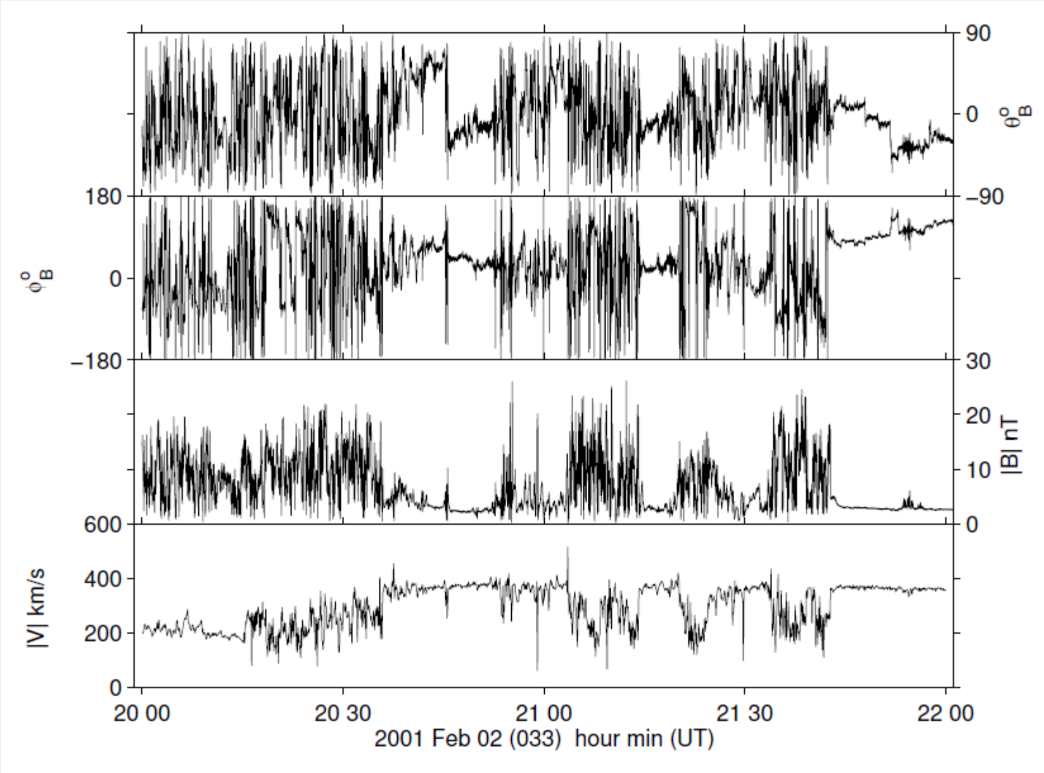
Quasi-Parallel Collisionless Shock – How?

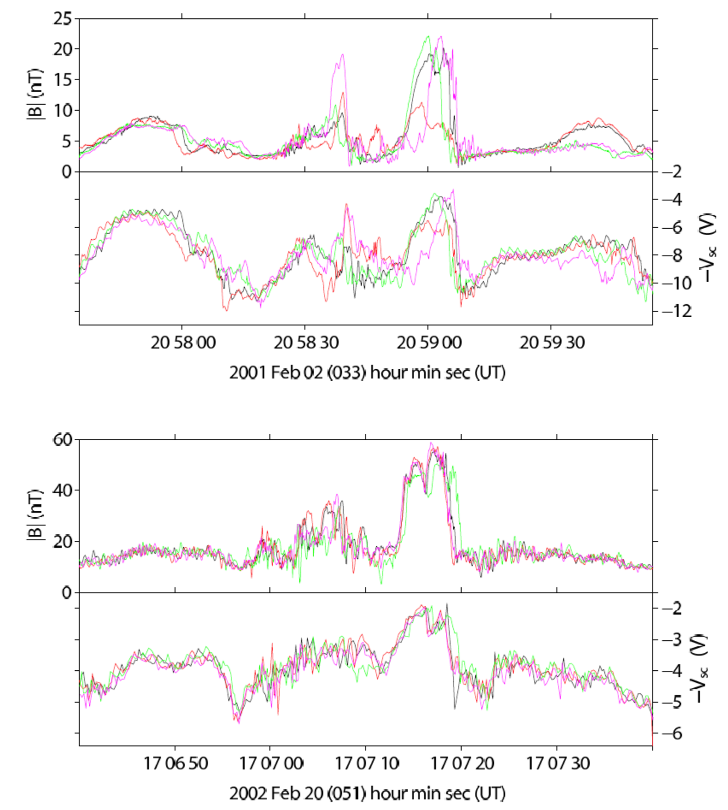


Foreshock-shock interaction

Earth’s Quasi-parallel Bow Shock

Presence of upstream energetic particles and upstream (and downstream) turbulence



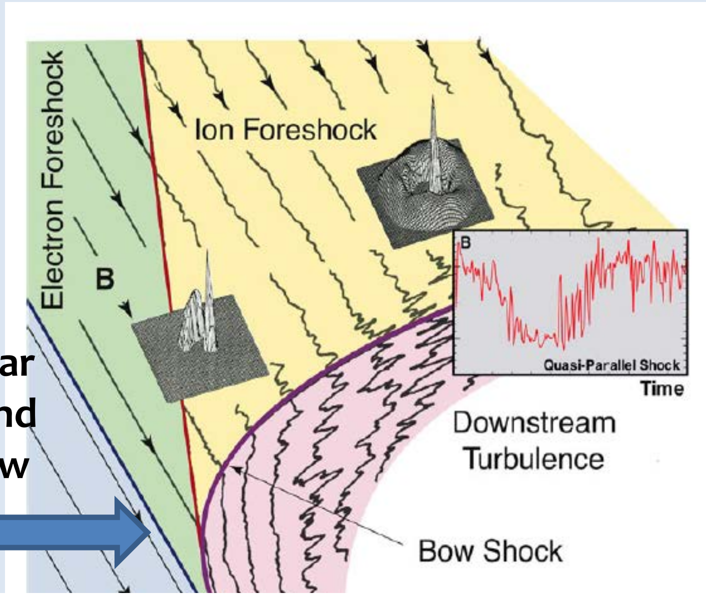


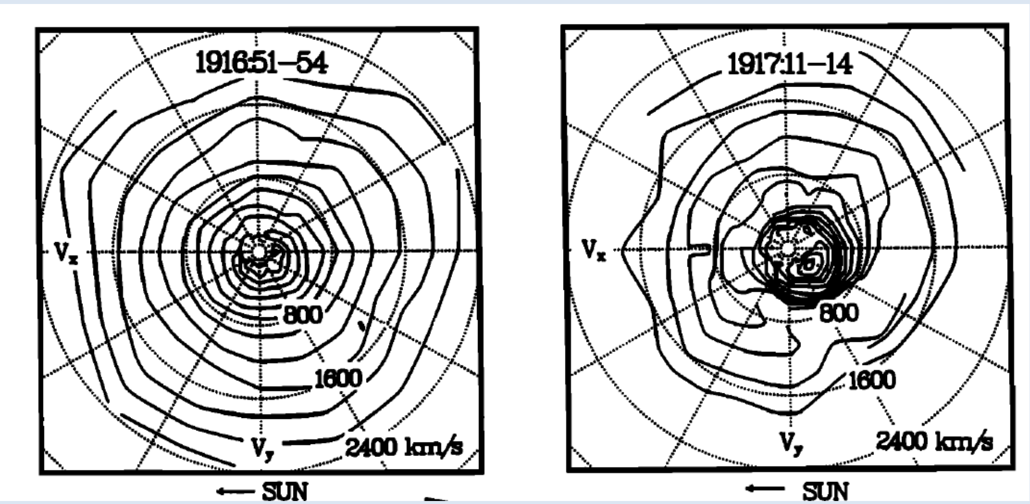
Large amplitude magnetic pulsations

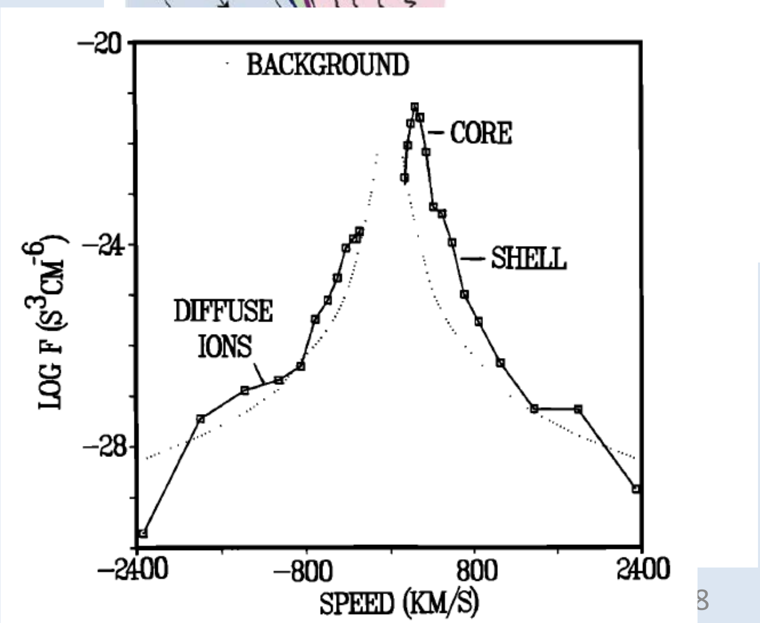
Earth’s quasi-parallel bow shock

- Observations consistent with simulations of shock structure
  - Time variability (reformation)
  - Large amplitude magnetic pulsations
  - Mixture of observed downstream ion distribution types (hot less dense vs denser cooler)

Solar Wind Flow



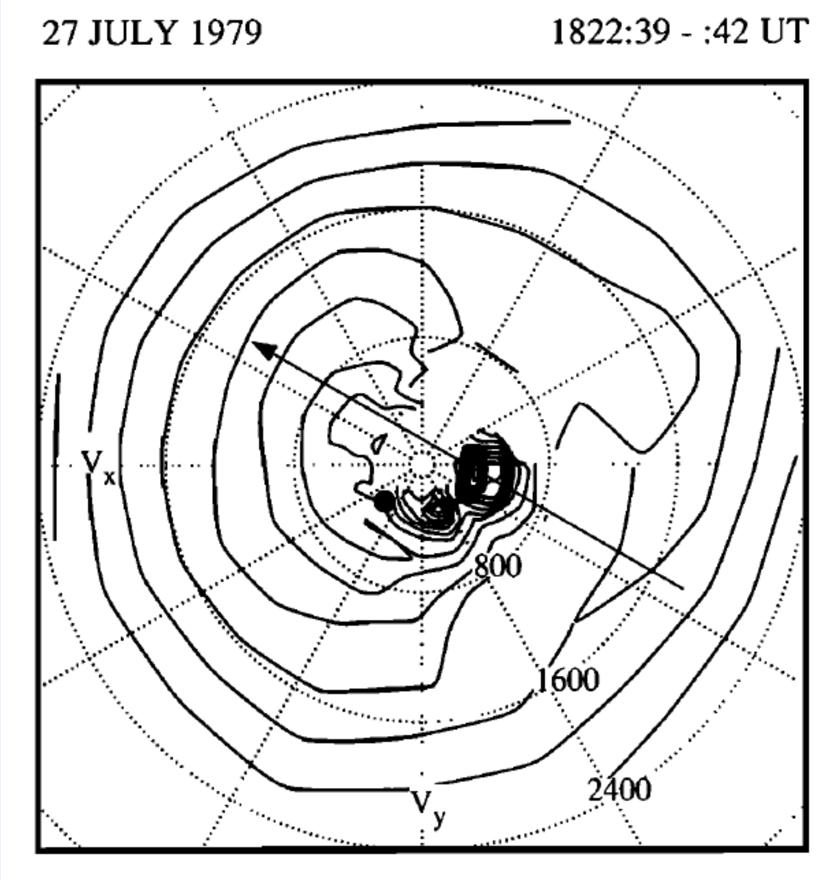




Thomsen et al 1990,
Gosling et al 1989

“Coherent” (specular) reflection at quasi-parallel shock

- Early ISEE observations of “coherent” beams consistent with specular reflection
- “2D” particle instrument – limited resolution in velocity space



Onsager et al 1990

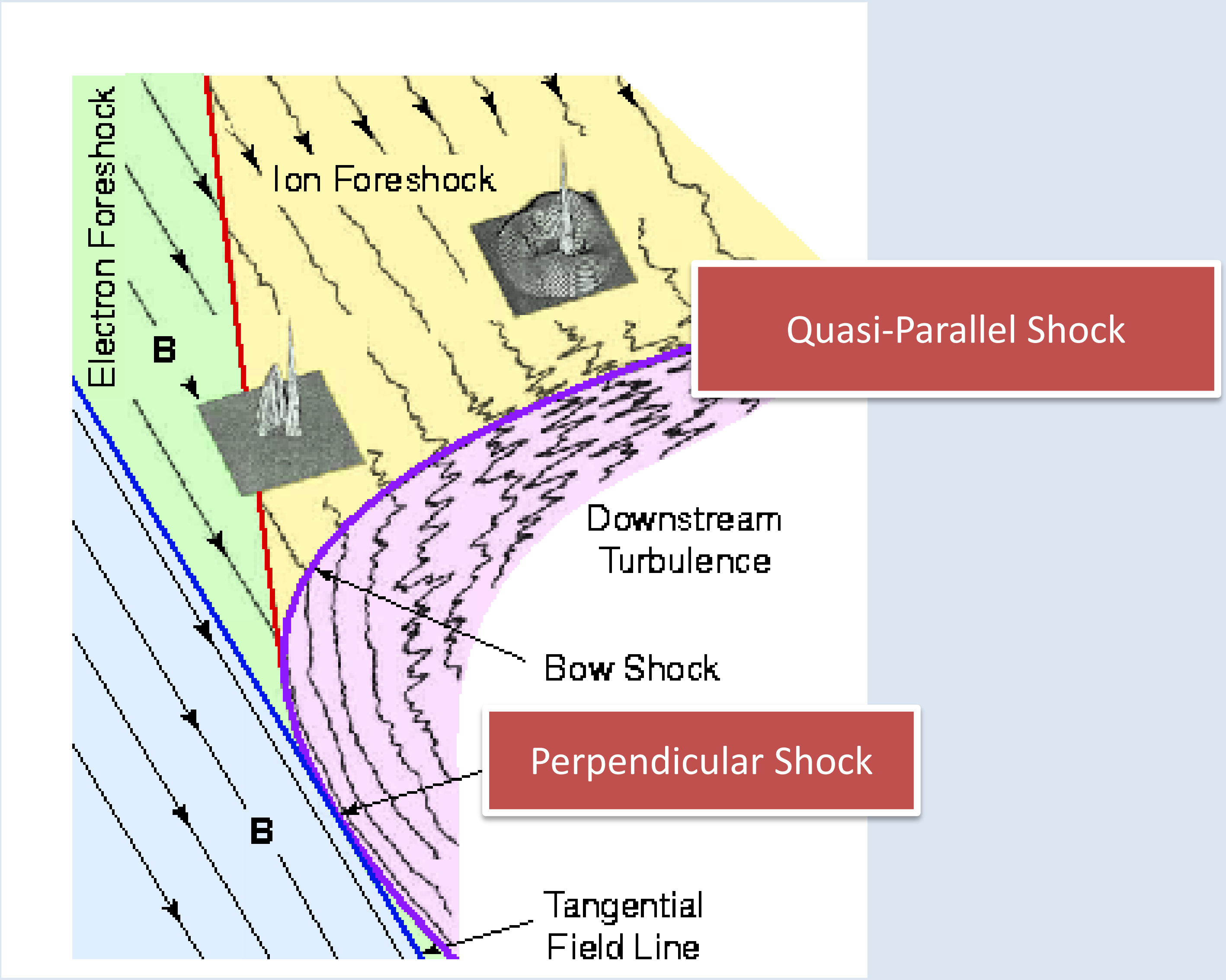
PARTICLE ACCELERATION AT SHOCKS

In a collisionless plasma

➢ *a small number of particles can reach large energy*

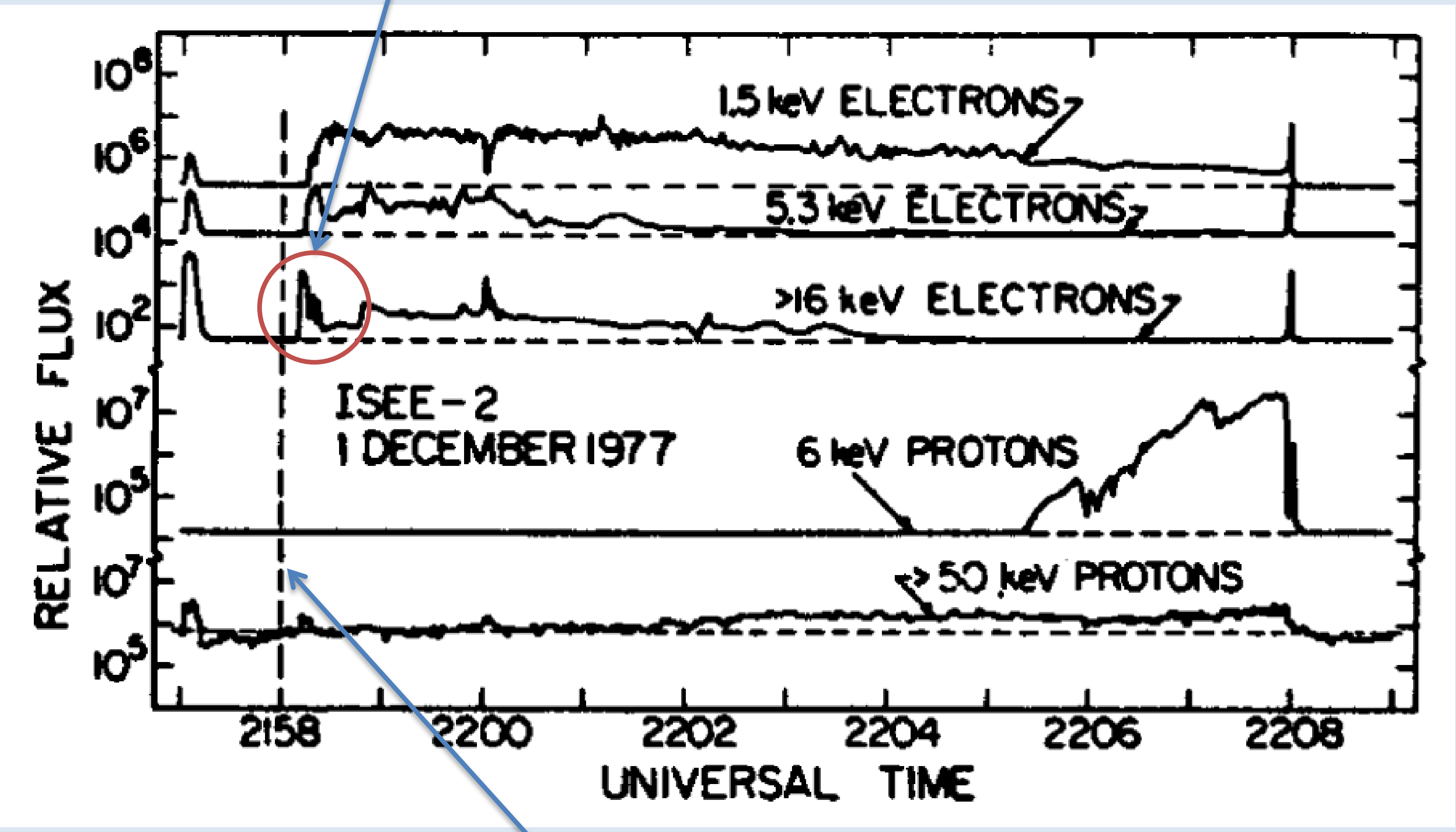
- Particle acceleration requires an electric field
- Electric fields arise from relative fluid motions
- Either:
  - Waves relative motion to background
  - Upstream flow relative to downstream
- Energy transferred directly from fluid motions to particles

The Earth's Bow Shock



solar wind
300-600 km/s

Electron Acceleration at the Quasi-Perpendicular Bow Shock



Upstream edge of the foreshock

Anderson et al. 1979

Electron acceleration at the foreshock edge (quasi-perpendicular shock)

Adiabatic motion of electrons in HT frame:

$$\mu = \frac{m(v^{HT})^2}{2B} = \text{const.}$$

Liouville's theorem: phase space density at point in velocity space is same as incident distribution at point whence reflected particles originated

$$f_r(-v_{||}^{HT}, v_{\perp}^{HT}) = f_0(v_{||}^{HT}, v_{\perp}^{HT})$$

But: there is normal electric field (not removed by transformation in HT frame).

Thus energy equation

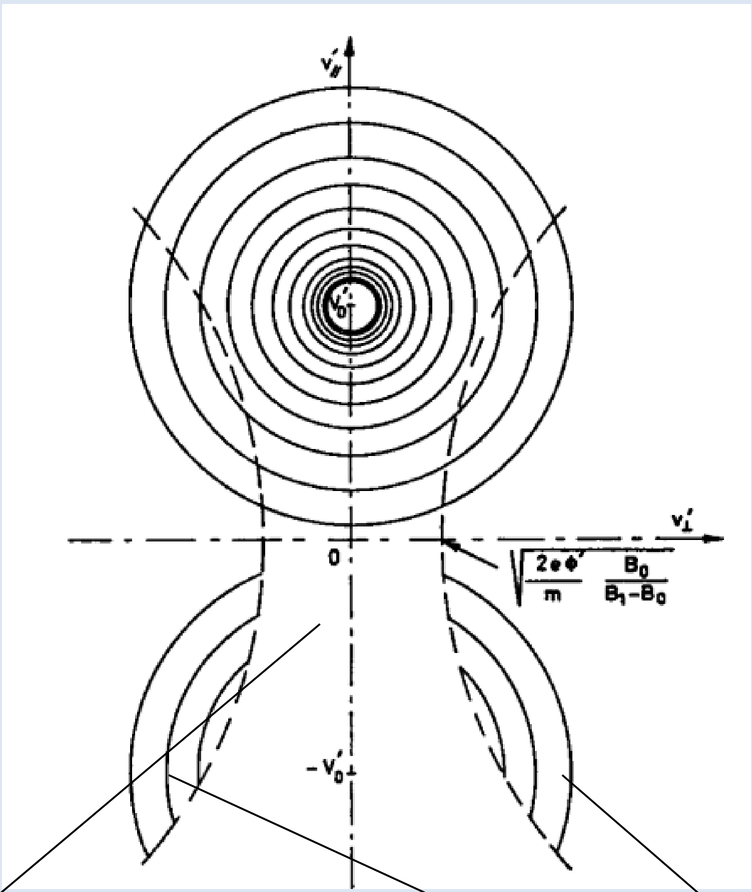
$$\frac{1}{2}m[v_{\|}^{HT}(x_i)]^2 = \frac{1}{2}m[v_{\|}^{HT}(x)]^2 + \Psi(x)$$

x_i=initial position upstream and Ψ pseudo-potential ($\phi^{HT}(x)$ electrostatic potential in the HTF)

$$\Psi(x) = \mu[B(x) - B(x_i)] - e\phi^{HT}(x)$$

Effect of potential: lower energy particles that would have been reflected, pass downstream

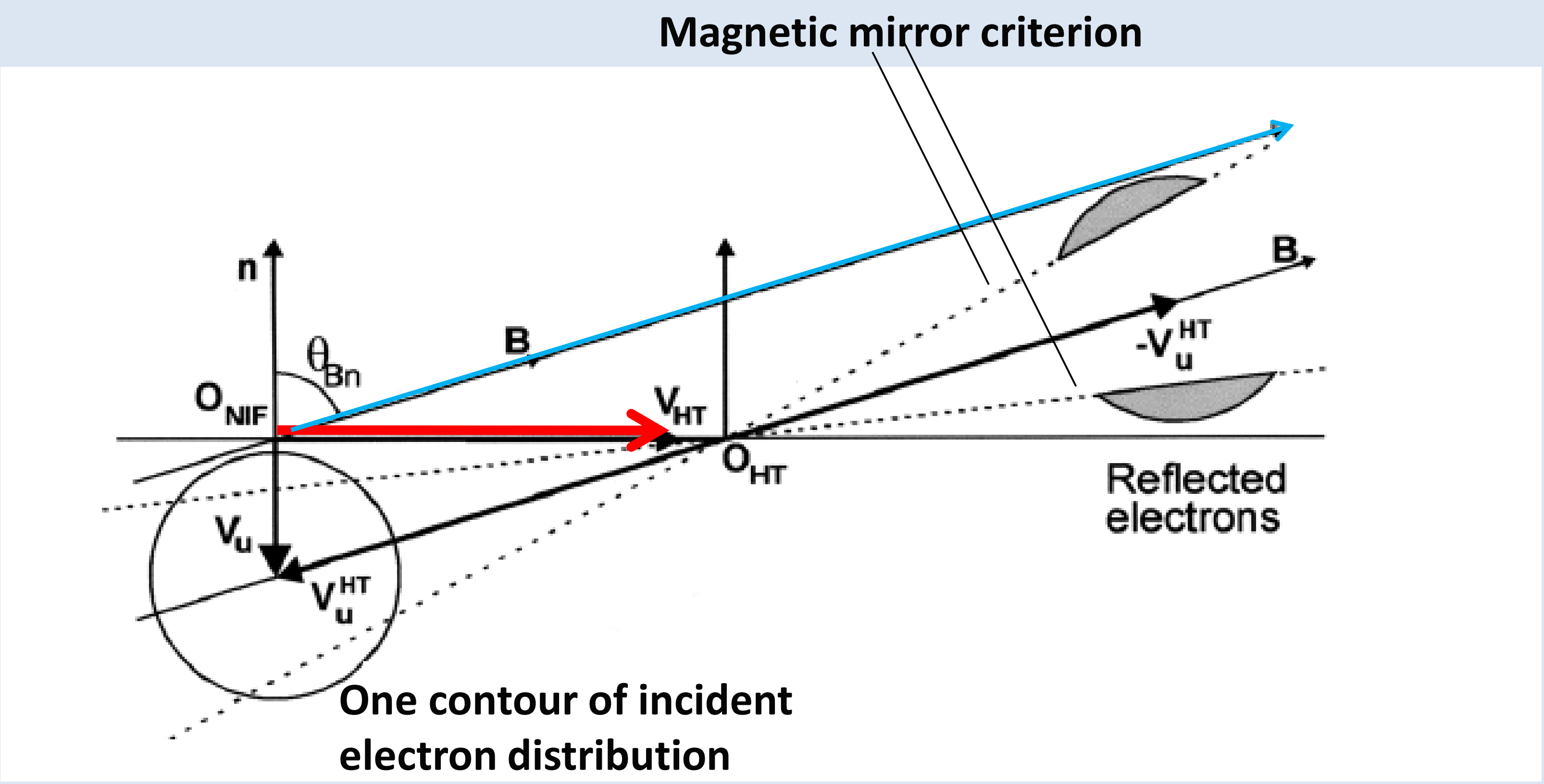
Reflected electrons: Ring beam with sharp edges



Larger loss cone due to cross-shock potential

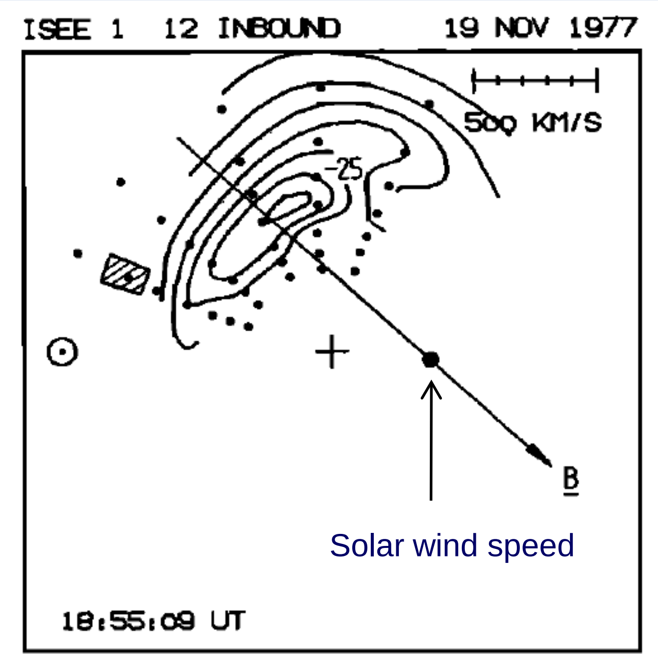
Portion reflected by magnetic mirroring modified by effect of cross-shock potential

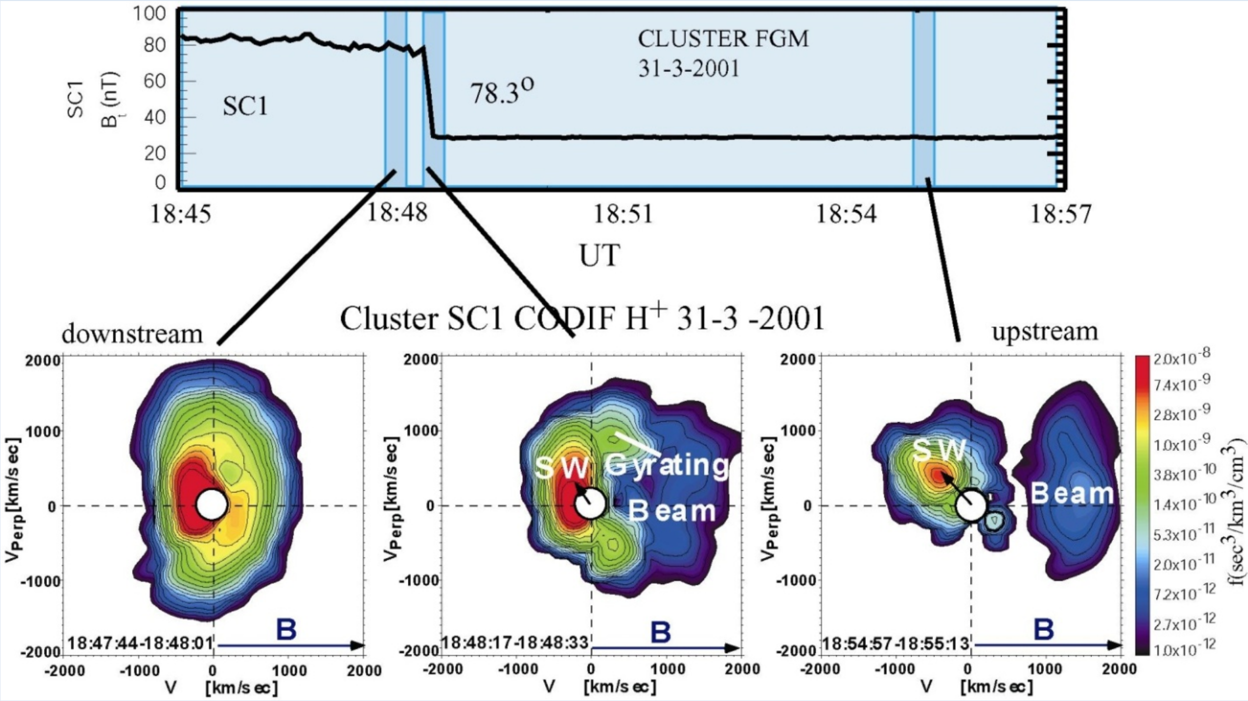
Leroy and Mangeney 1984



As V_{HT} increases (Θ_{Bn} closer to 90°) portion that mirrors comes increasingly from outer part of incident distribution: reflected density depends strongly on distribution function above thermal energies (non-Maxwellian tail).

Field –aligned beams (FABs) upstream of the quasi-perpendicular shock





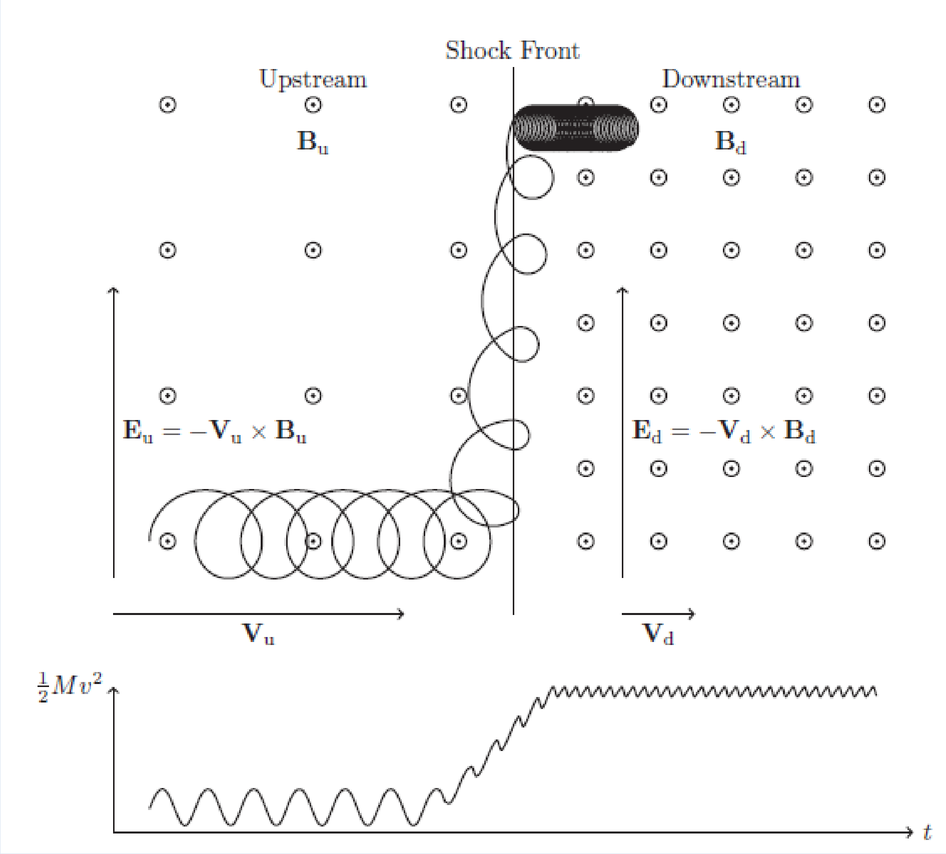
Paschmann et al. 1981

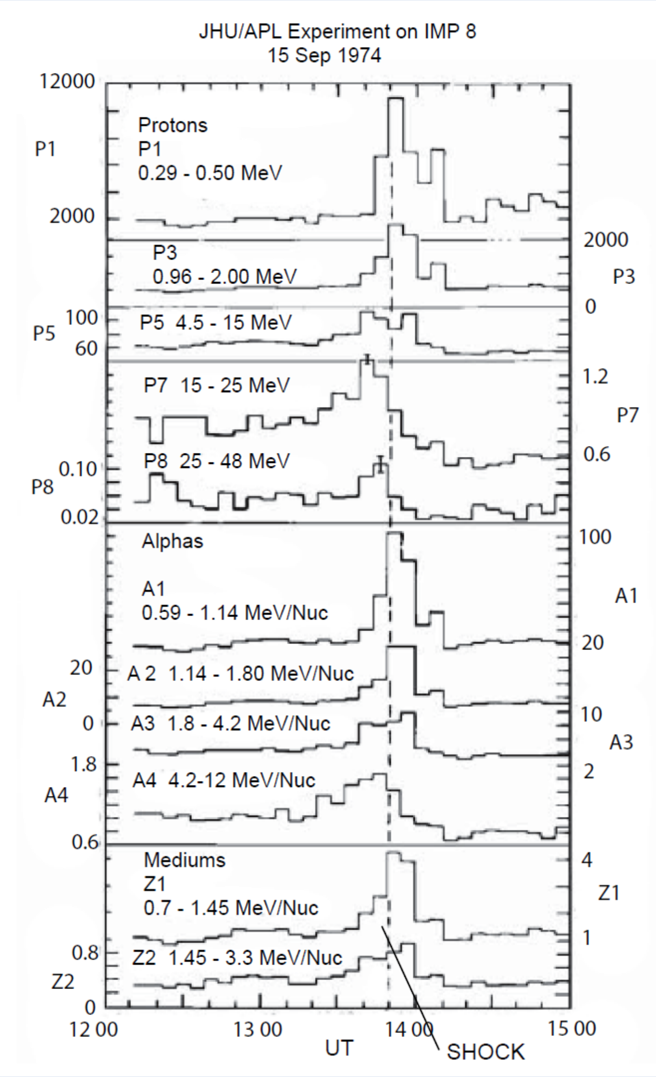
Kuchaek et al. 2004

SHOCK DRIFT ACCELERATION - SDA

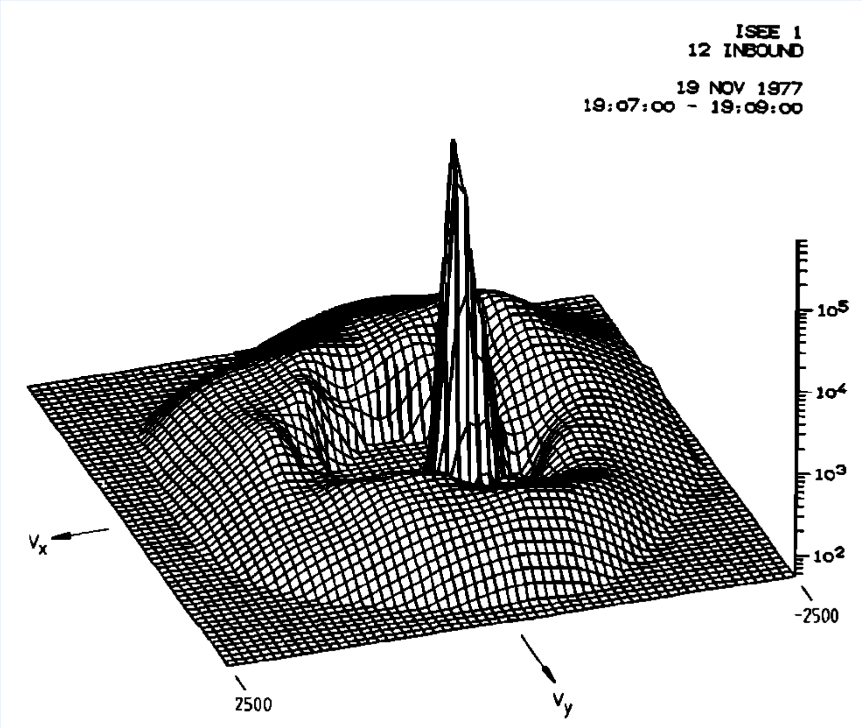
Energy gain from motional electric field …

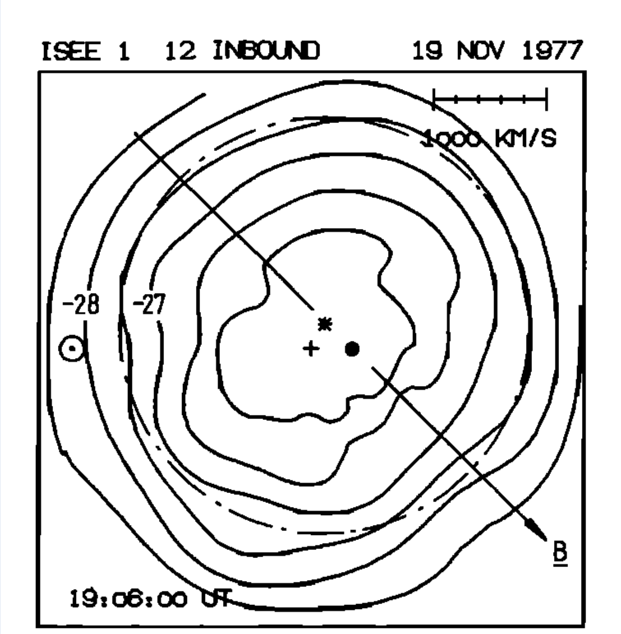
- if a particle can be held in shock frame
- and drifts in direction of electric field





Diffuse ions upstream of Earth's quasi-parallel bow shock

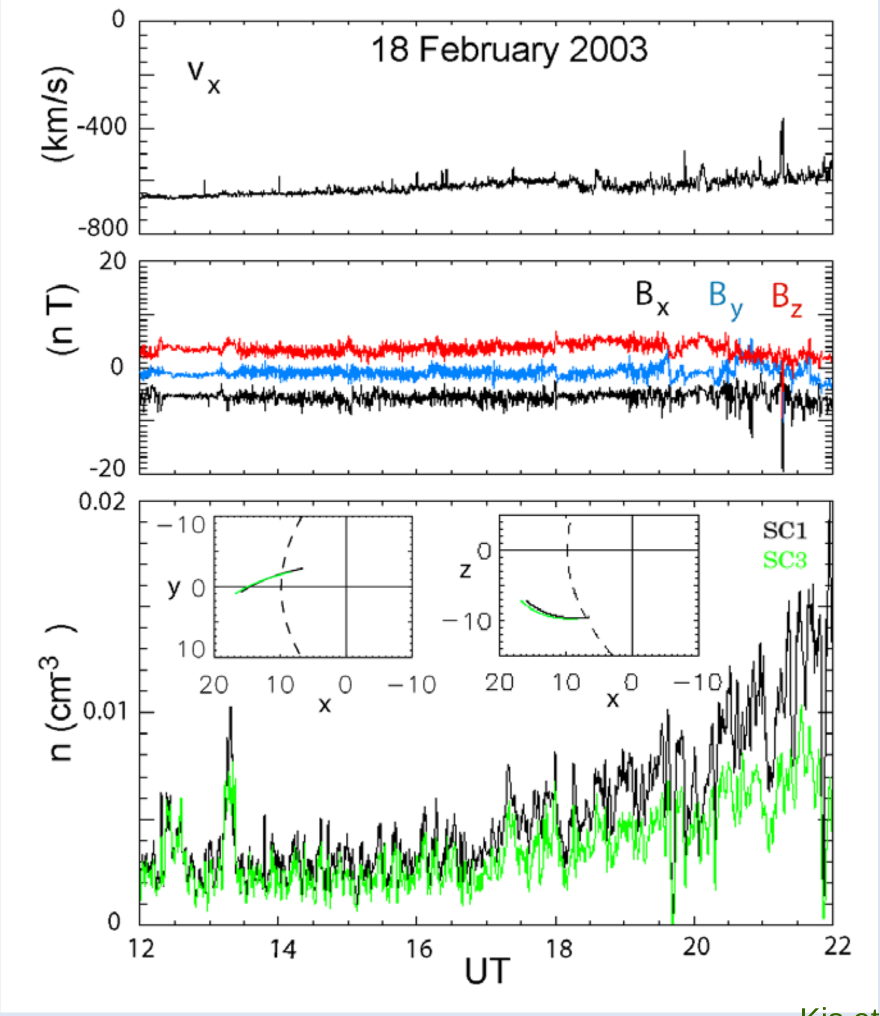


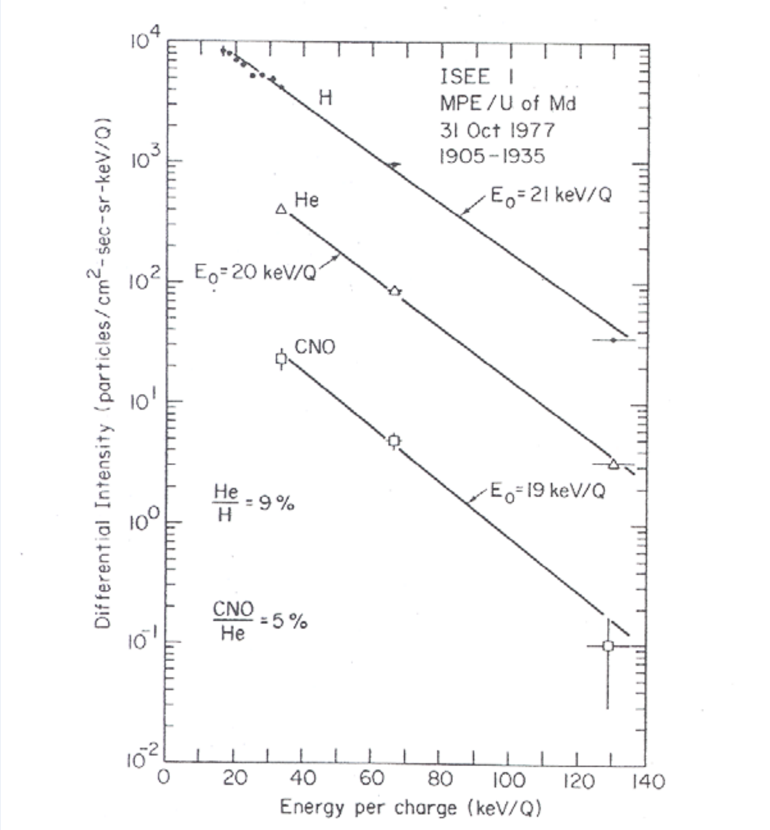


Paschmann et al. 1981

Observations of Diffuse Ions at Bow Shock

Particle density at 24 – 32 keV at two spacecraft, SC1 (black) and SC3 (green). SC3 was about 1.5 Re closer to the bow shock .



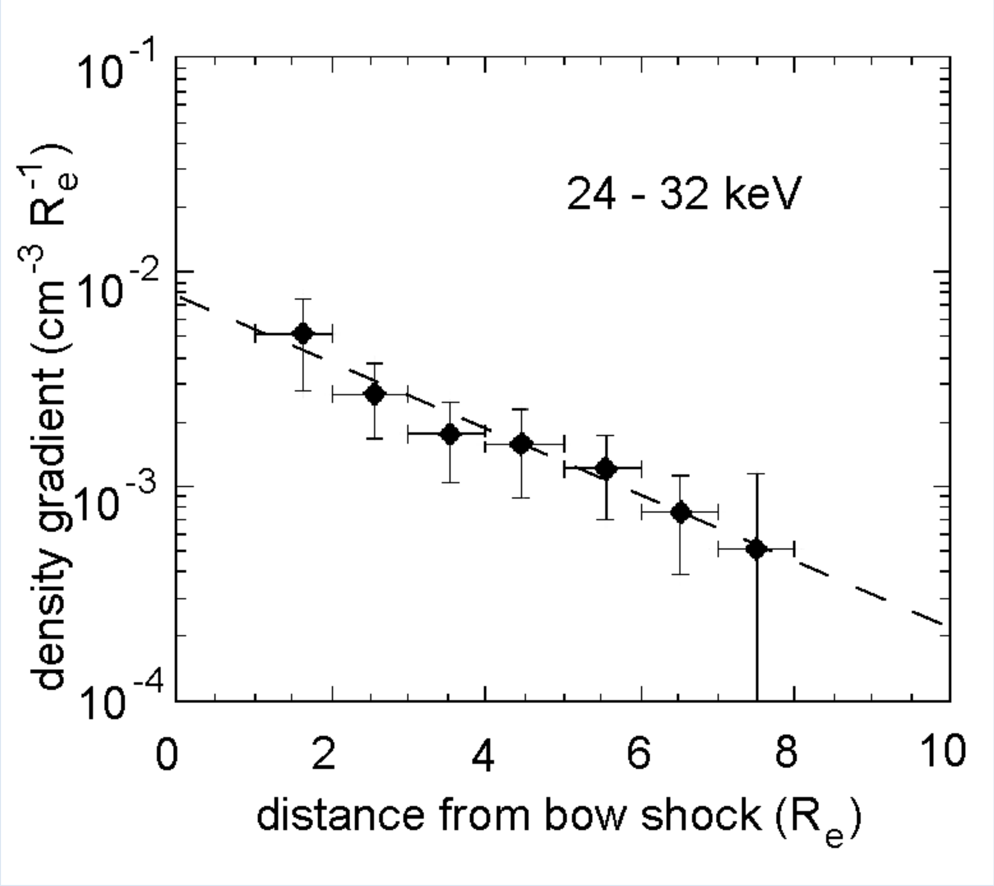


Log(flux) versus energy/charge (linear)

Spectra are exponentials in energy/charge with the same e-folding energy for all species

Ipavich et al. 1981

Kis et al. 2004



Kis et al. 2004

The gradient of upstream ion density in the energy range 24-32 keV as a function of distance from the bow shock (Cluster) in a lin vs log representation.

The gradient (and the density itself) falls off exponentially.

Strong indication for diffusive transport in the upstream region:
Downstream convection is balanced by upstream diffusion

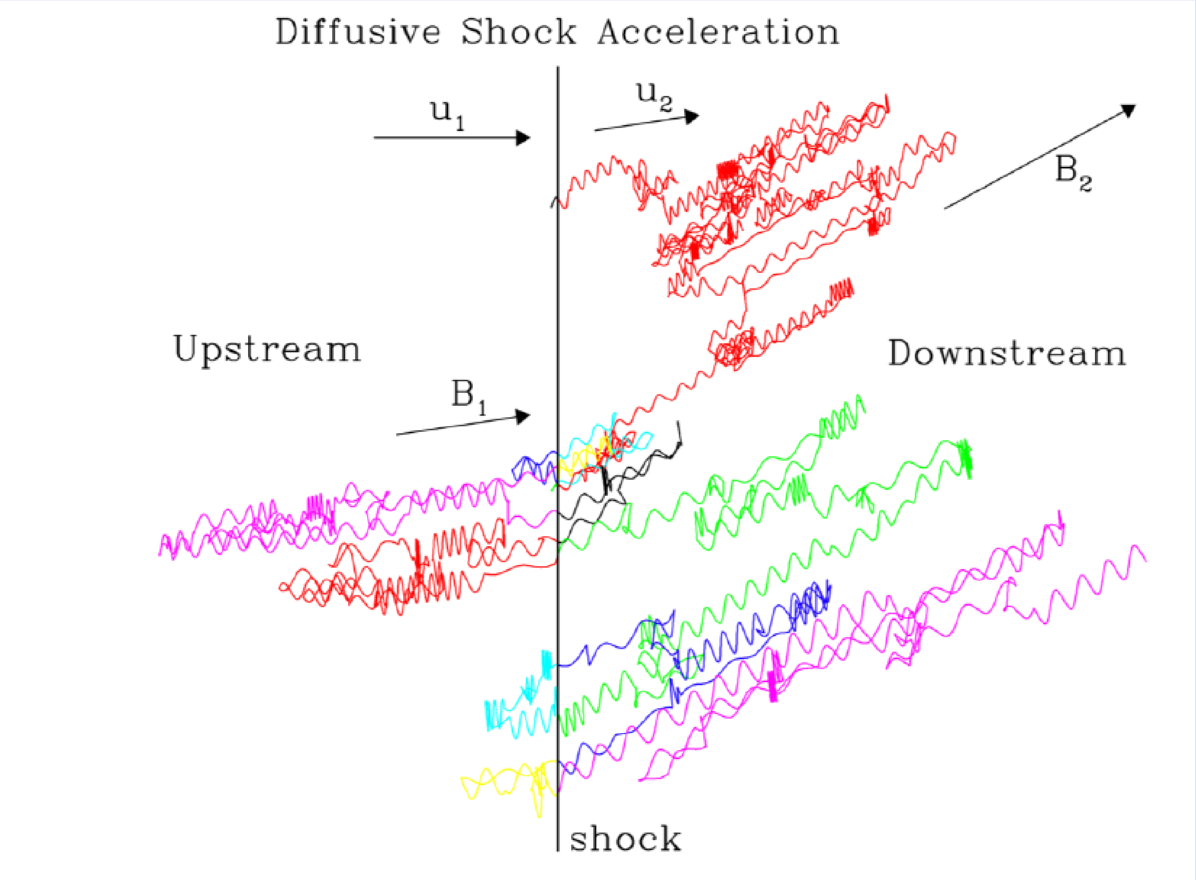
Diffusive Shock Acceleration - DSA

The Parker transport equation for energetic particles

- Diffusion
- Convection
- Adiabatic deceleration

Monte Carlo solution (Ellison)

- Assume scattering law for particles
- Follow particles in given flow profile
- Can include modified flow profile



Baring 2013

The Parker transport equation for the phase space density f may be written as the sum of various physical effects, as indicated:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i}[\kappa_{ij}\frac{\partial f}{\partial x_j}] \qquad \text{(diffusion)}$$

$$-U_i\frac{\partial f}{\partial x_i} \qquad \text{(convection)}$$

$$+\frac{1}{3}\frac{\partial U_i}{\partial x_i}[\frac{\partial f}{\partial \ln p}] \qquad \text{(energy change)}$$

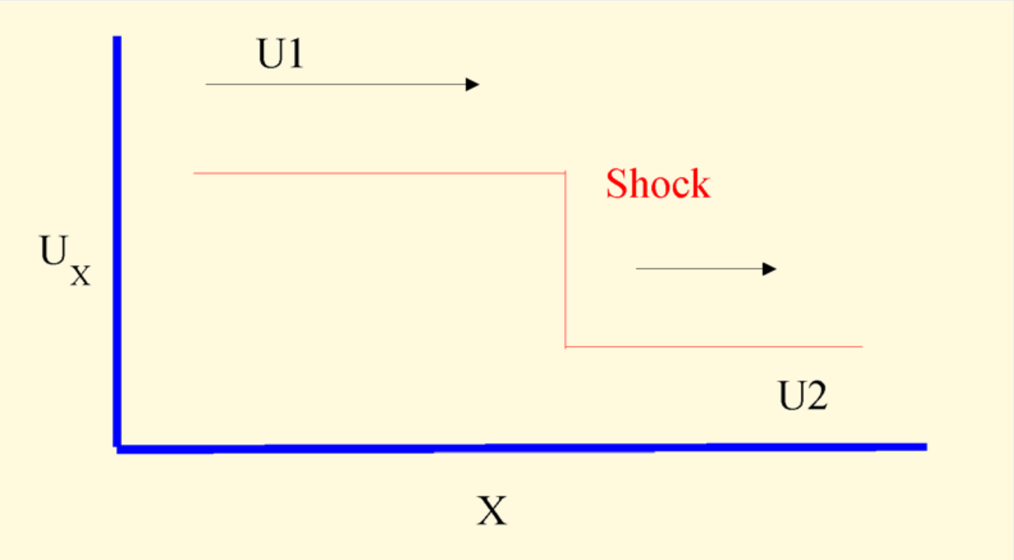
$$+Q(x_i, t, p) \qquad \text{(source term)}$$

This equation contains both spatial transport and acceleration. Statistical acceleration can be incorporated by adding momentum diffusion

$$\frac{1}{p^2}\frac{\partial}{\partial p}[p^2 D_{pp}\frac{\partial f}{\partial p}]$$

Consider a one-dimensional flow $U_x(x)$ as shown.

(the shock ratio $U_1 / U_2 < 4$)



Solve the Parker transport equation for boundary condition

$$\frac{\partial U}{\partial x} = (U_2 - U_1)\delta(x)$$

$$f_1(x=0) = f_2(x=0)$$

DSA: Convection-Diffusion

Omnidirectional distribution function: $f(\mathbf{x}, p)$

Convection-diffusion (Parker) equation in conservation form:

$$\frac{\partial f}{\partial t} + \nabla \cdot \mathbf{S} + \frac{1}{3}\frac{1}{p^2}\frac{\partial}{\partial p}[p^3 \mathbf{U} \cdot \nabla f] = 0,$$

Energetic particle streaming: $\mathbf{S} = \int \mathbf{v} f(\mathbf{p})\, d\boldsymbol{\Omega}$

$$\mathbf{S} = -\mathbf{K} \cdot \nabla f - \mathbf{U}\frac{p}{3}\frac{\partial f}{\partial p}.$$

Particle diffusion coefficients $\kappa_{\|}$ and $\kappa_{\perp}$ parallel and perpendicular to the magnetic field $\mathbf{B}$.

Shock: 1D planar discontinuity, upstream flow U_1, downstream U_2.

Diffusion coefficient normal to the shock, x:
$\kappa_{xx} = \kappa_{\|} \cos^2 \theta_{Bn} + \kappa_{\perp} \sin^2 \theta_{Bn}$

Planar shock in steady state:

$$\frac{\partial}{\partial x}\left[Uf - \kappa_{xx}\frac{\partial f}{\partial x}\right] = \frac{1}{3}\frac{\partial U}{\partial x}\frac{\partial f}{\partial \ln p}$$

Boundary condition at shock $x = 0$ due to velocity jump from U_1 to U_2:

$$\partial U/\partial x = (U_2 - U_1)\delta(x)$$

$$\begin{aligned} \text{Upstream:} \quad & f(x=-\infty, p) = f_1(p) \quad \text{(and given)} \\ \text{Downstream:} \quad & f(x \geq 0, p) = f_2(p) \quad \text{(independent of } x\text{)} \end{aligned}$$

Integrating from $x = -\infty$ to $x = +\infty$:

$$U_2 f_2(p) - U_1 f_1(p) = +\frac{1}{3}(U_2 - U_1)\frac{\partial}{\partial p}\left[p f_2(p)\right].$$

Solve as differential equation for the downstream distribution function $f_2(p)$, using shock compression ratio $r = U_1/U_2$:

$$p\frac{\partial f_2(p)}{\partial p} + \frac{r+2}{r-1} f_2(p) = \frac{3r}{r-1} f_1(p),$$

with the solution

$$f_2(p) = a p^{-\Gamma} \int_0^p f_1(p') p'^{\Gamma - 1} dp'$$

$$a = 3r/(r-1)$$
$$\Gamma = (r+2)/(r-1)$$

Suppose injection at low energy $f_1(p) \propto \delta(p_0)$, then f_2 has a power law form above p_0.

$$f_2(p) \propto p^{-\Gamma}$$

Upstream $x < 0$:

$$f(p) = f_1(p) + (f_2(p) - f_1(p)) \exp\left(U_1 \int_x^0 \frac{dx'}{\kappa_{xx}(x', p)} \right).$$

Upstream:

- Distribution above p_0 falls off exponentially from the shock with an e-folding distance $L_e = x_0 = U_1/\kappa_{xx}$.
- Dependence of f on x upstream of the shock is determined by the functional form of the parallel diffusion coefficient.

Downstream:

- Downstream spectral exponent Γ depends *solely* on the shock compression ratio, r.
- For a strong shock r approaches 4, and the lowest value of the spectral index of the distribution function in scalar momentum is $\Gamma = 2$.

Alternative derivation (Bell approach – Drury, 1983)

The PLAN:

- for energetic particle crossing upstream to downstream: derive probability that never returns to the shock
- for particle returning to shock: find increase in momentum
- find probability that particle will reach a given momentum
- construct differential momentum distribution

Let n be number of particles with velocity v (local fluid frame)

- flux of particles escaping downstream to infinity is nU_2
- flux of particles from upstream into downstream is $F = \int_0^1 \mu v n \, d\mu / 2$ (μ is the cosine of the pitch angle)
- For an almost isotropic distribution: $F = nv/4$
- The probability of particles **not** returning to shock: $nU_2/(nv/4) = 4U_2/v$.

For particle crossing upstream $\rightarrow$ downstream:
average change per particle in momentum per unit time:

$$< \Delta p >= p \int_0^1 [\mu(U_1 - U_2)/v] 2\mu \, d\mu = \frac{2}{3} p(U_2 - U_1)/v.$$

(And same result crossing other way.)

Number density of particles N with momentum larger than p

$$N(x,p) = \int_p^\infty f(x,p')dp'.$$

Assume all particles advected from upstream to the shock have same momentum p_0. Then upstream

$$N(-\infty,p) = N_1(p) = \begin{cases} 0 & p > p_0 \\ N_0 & p < p_0 \end{cases},$$

and downstream

$$N(\infty,p) = N_2(p) = \frac{U_1}{U_2}N_0, \qquad p < p_0.$$

Let p_n and v_n be the momentum and velocity of a particle which returns from the downstream region n times and thus makes a total of $2n$ crossings of the shock. To first order in U/v the momentum is given by

$$p_n \sim \prod_{i=1}^{n} \left[1 + \frac{4}{3}(U_1 - U_2)/v_i \right] p_0,$$

so that

$$\ln(p_n/p_0) = (4/3)(U_1 - U_2) \sum_{i=1}^{n} \frac{1}{v_i}.$$

The probability of the particle returning n times from downstream to upstream is

$$P_n \sim \prod_{i=1}^{n} \left(1 - \frac{4U_2}{v_i} \right) \Rightarrow \ln P_n \sim -4U_2 \sum_{i=1}^{n} \frac{1}{v_i} = -3 \frac{U_2}{U_1 - U_2} \ln(p_n/p_0)$$

So. . .

$$P_n = (p_n/p_0)^{3U_2/(U_1-U_2)}.$$

The number density of particles accelerated to momentum p_n is the total number density of particles, $N_2(p_0)$, multiplied by probability P_n of crossing the shock sufficiently often.

$$N_2(p_n) = P_n N_2(p_0) = \frac{U_1}{U_2}\left(\frac{p_n}{p_0}\right)^{-3U_2/(U_1-U_2)} N_0, \qquad p_n > p_0,$$

and

$$f_2(p) = -\frac{\partial N_2}{\partial p} = \frac{N_0}{p_0}\frac{3U_1}{U_1-U_2}\left(\frac{p}{p_0}\right)^{-(U_1+2U_2)/(U_1-U_2)}, \qquad p_n > p_0,$$

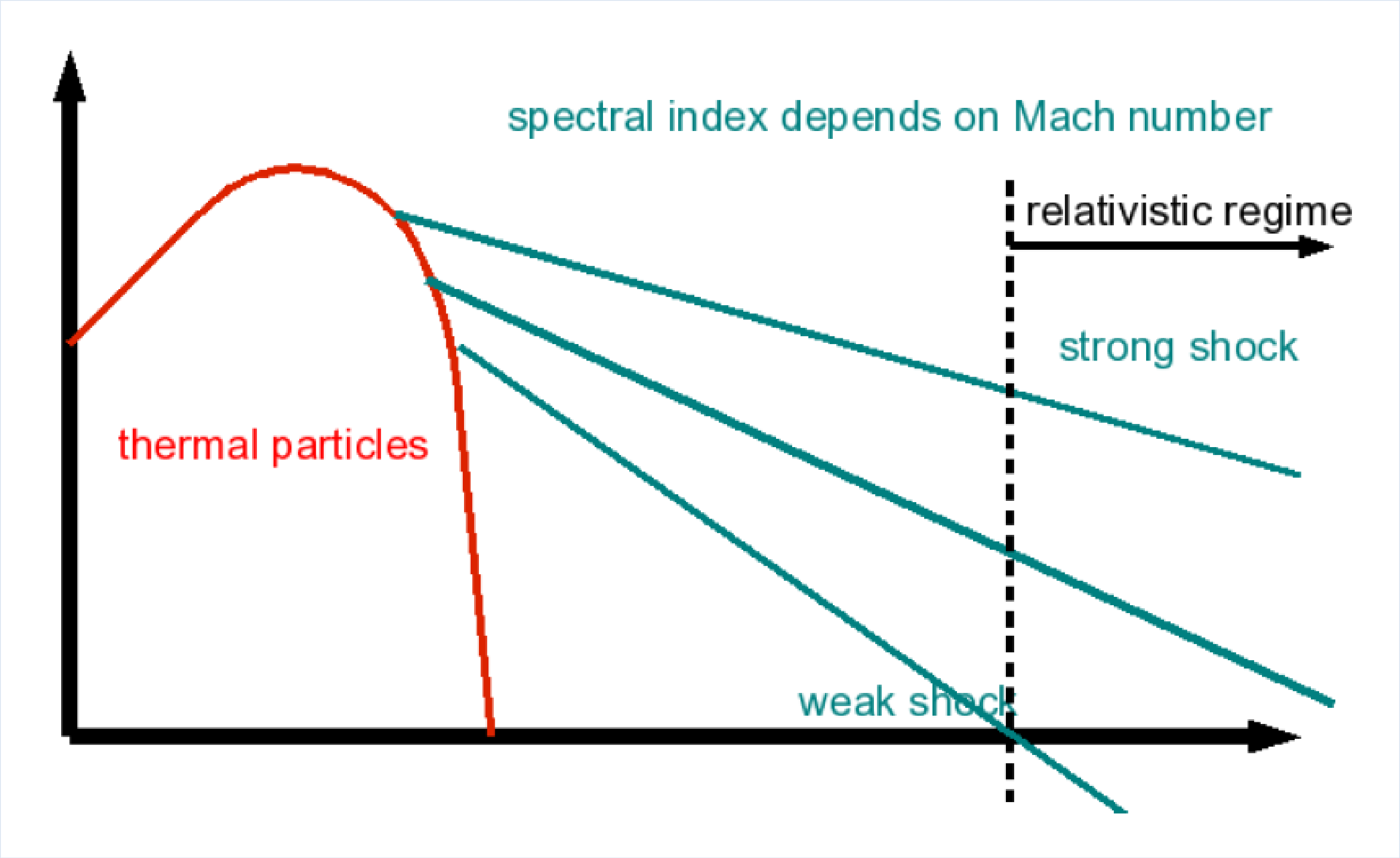
(Same as for particle transport equation.)

To obtain a power law . . .

- the momentum gained by a particle is proportional to its momentum
- the momentum gained by a particle is proportional to its probability of escaping from the shock

The constant of proportionality is determined by the upstream and downstream fluid velocities, U_1 and U_2, and hence they determine the slope of the power law.

DSA – Power Law

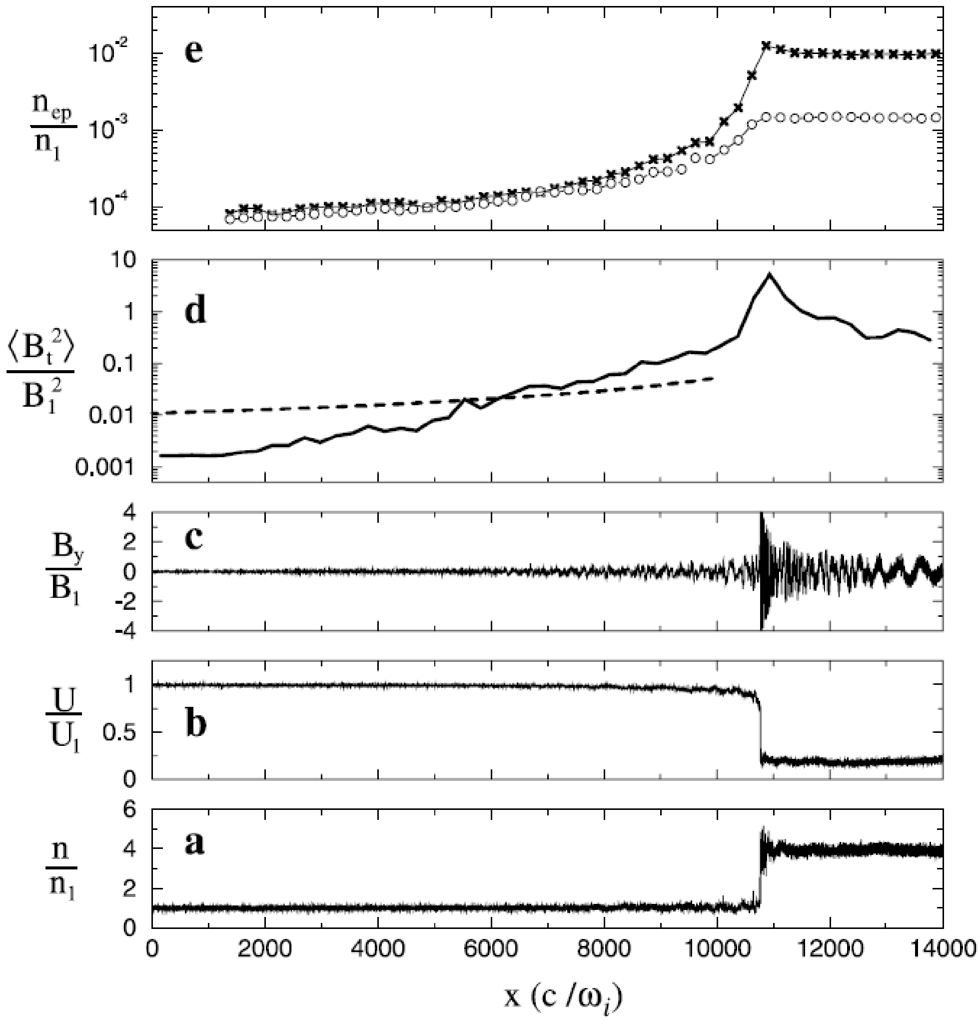


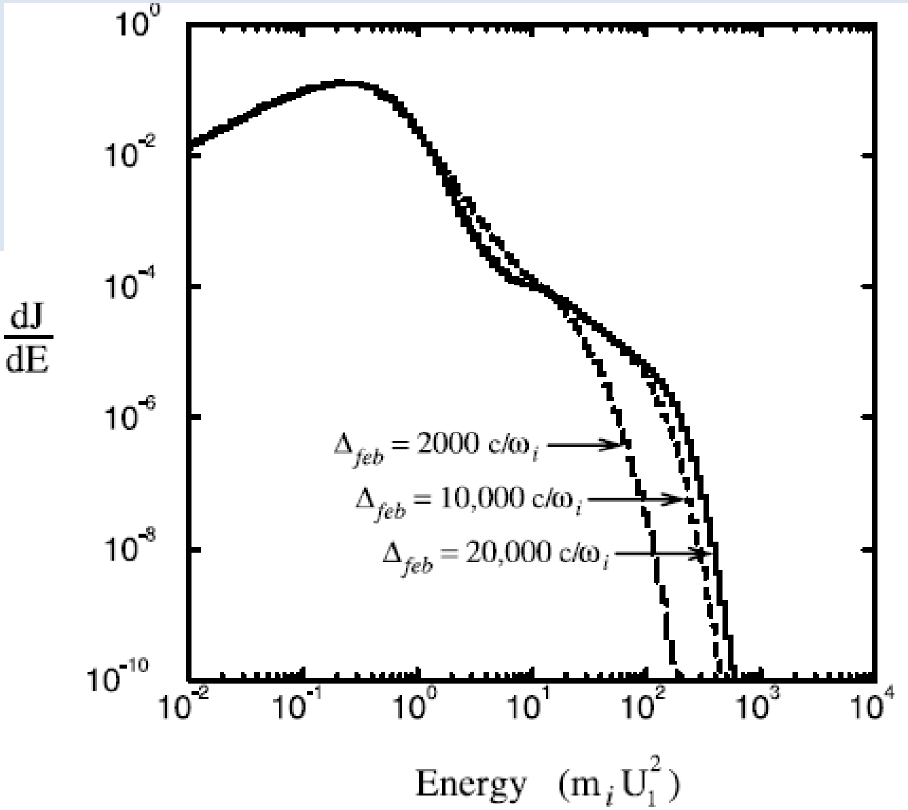
Kaiser 2007

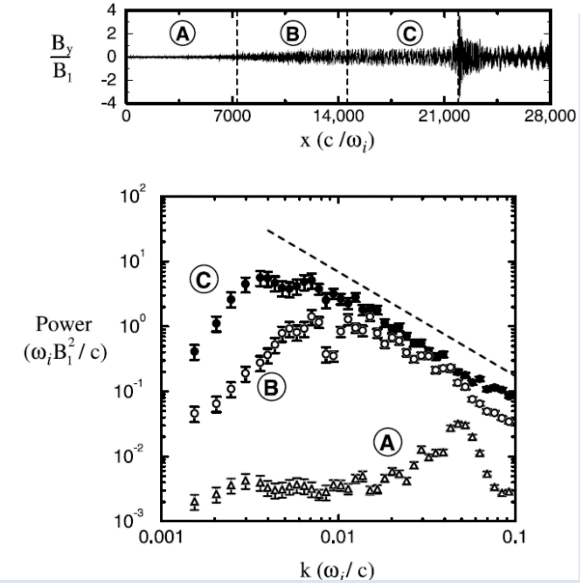
DSA – Other Issues

- Time evolution
  - Time scale (rate of acceleration) depends on upstream diffusion coefficient
- Efficiency and shock geometry (quasi-perp vs quasi-parallel)
- Escape and propagation
- Global aspects
  - 2D and 3D plasma physics
  - Curved shocks

Simulating Diffusive Acceleration

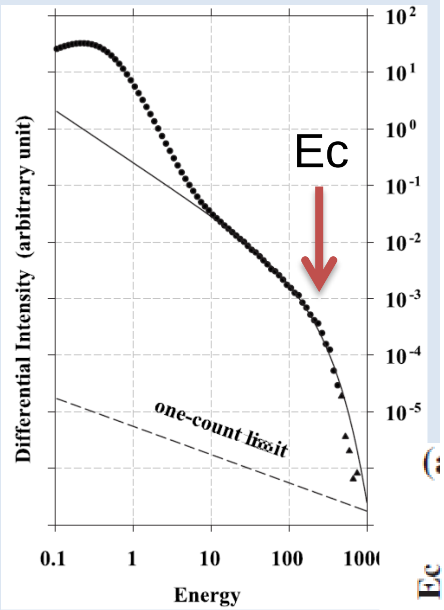


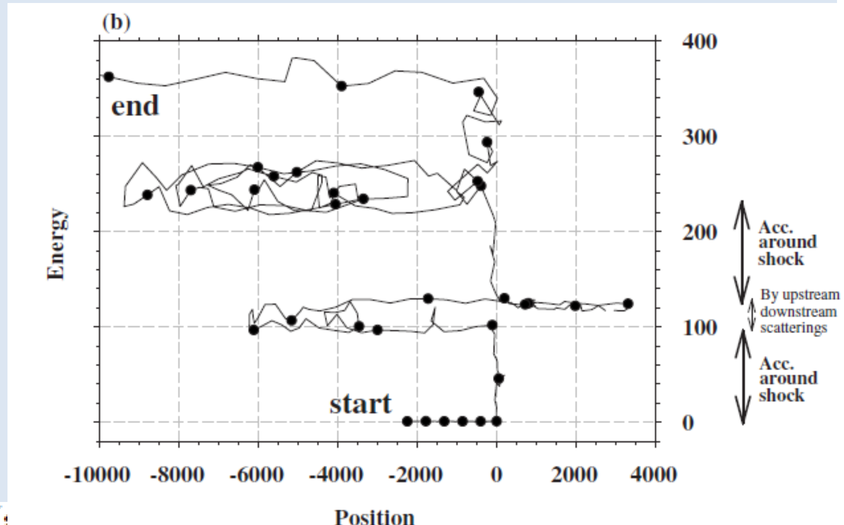


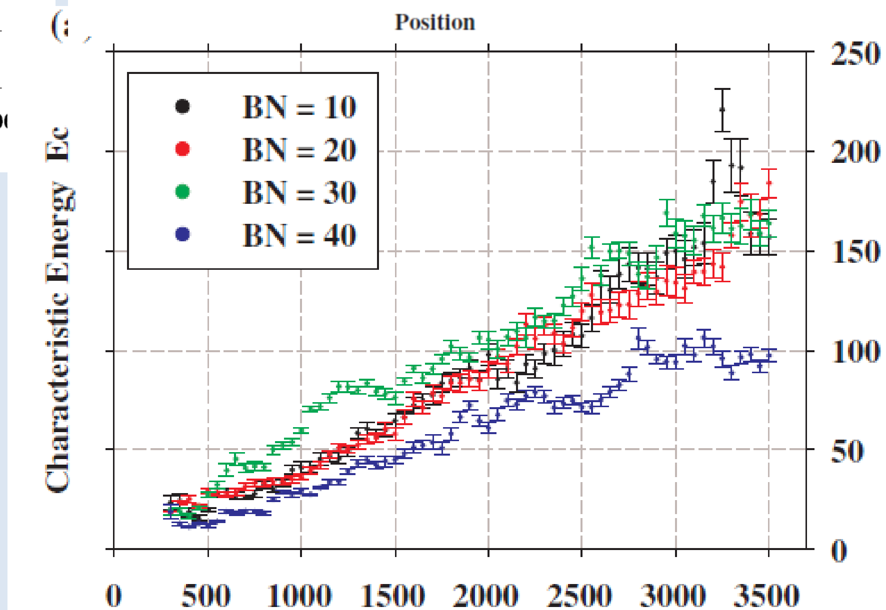


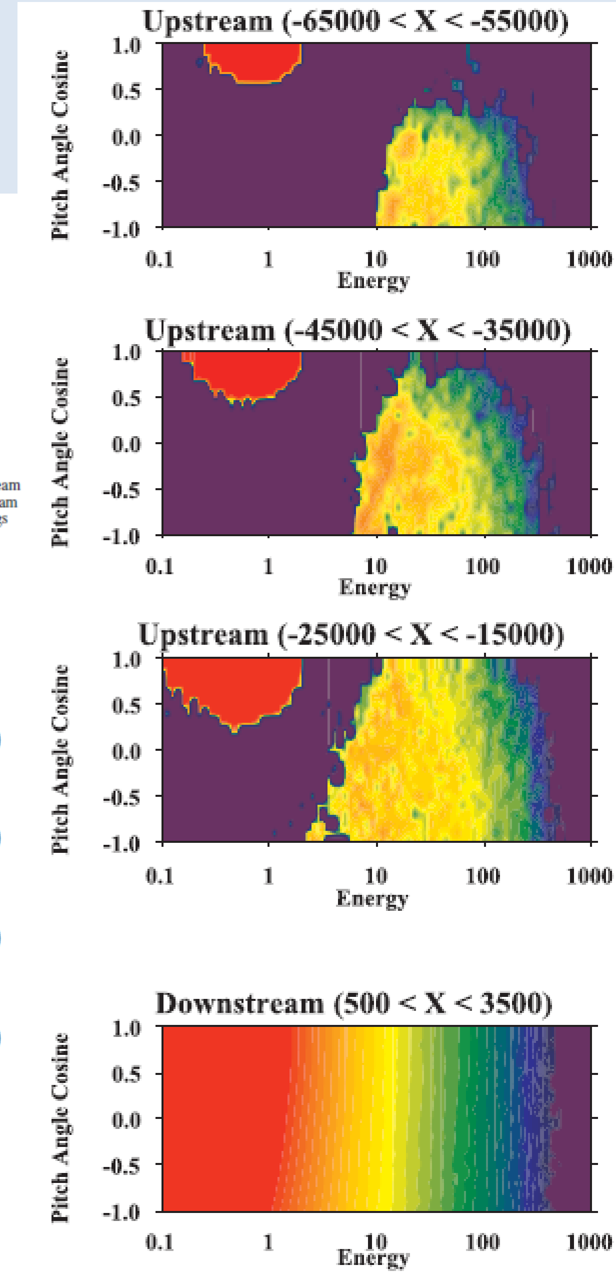
Giacalone 2004

Simulating Diffusive Acceleration









Sugiyama 2011

The Injection "Problem"

• The "problem" of "injecting" thermal particles to the higher energies of diffusive acceleration

• If shock structure assumed independent of accelerated population – some initial distribution of energetic ions required

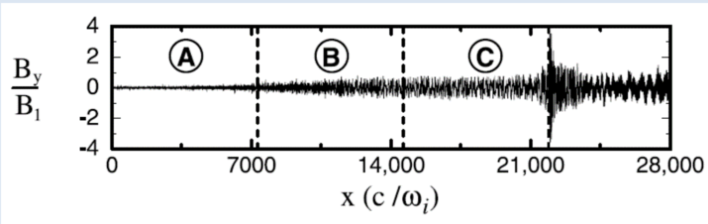
• "Injection" via heating or assumed scattering and feedback on shock structure (Monte Carlo, nonlinear DSA).

The injection "problem" reformulated by simulationists:

Can energetic ions be extracted from the thermal population?

The Injection "Problem"

- ***Hybrid simulations of parallel and quasi-parallel shocks***



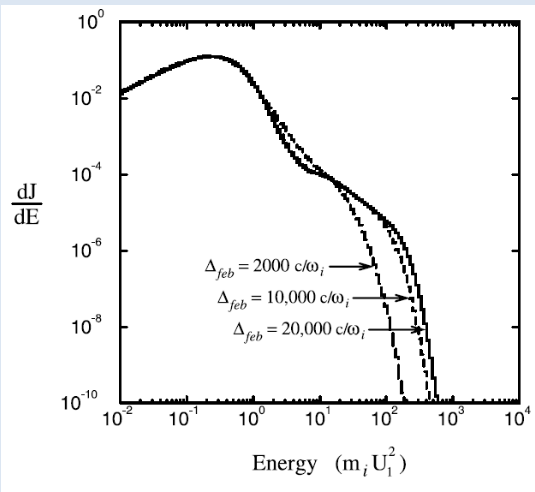
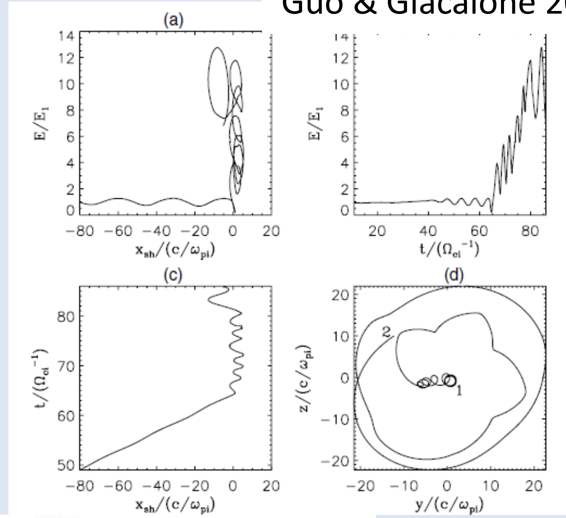


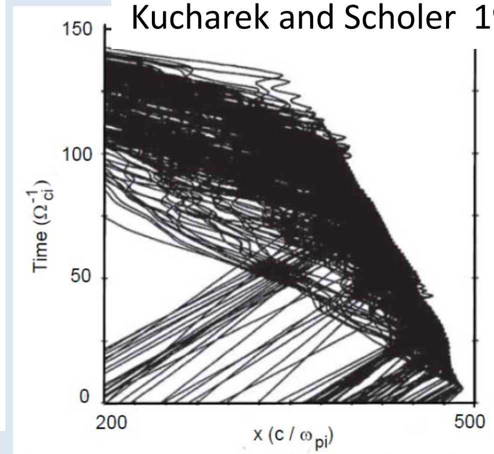
Fig. 2.—Simulated downstream energy spectra for all runs excluding run 2. All spectra were obtained near the end of each respective simulation, at which point the distribution was no longer evolving with time.

Giacalone 2004

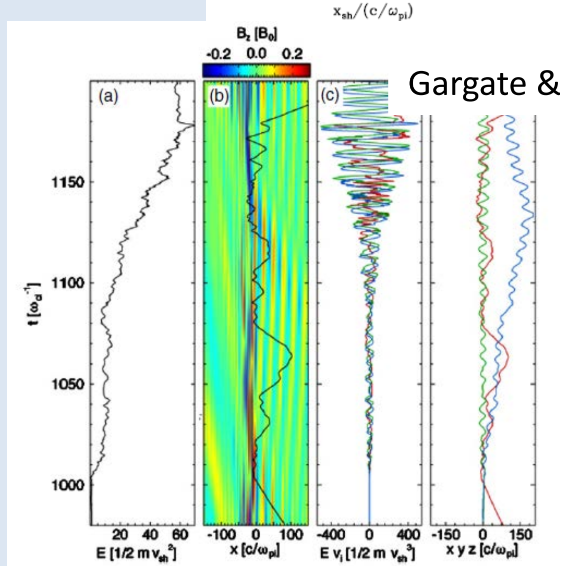
Guo & Giacalone 2013



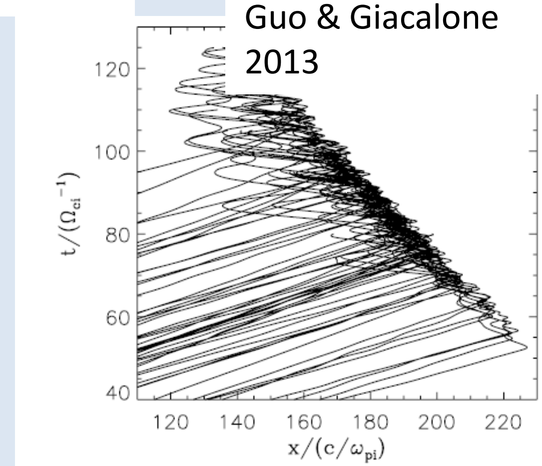
Kucharek and Scholer 1991



Gargate & Spitkovsky 2012



Guo & Giacalone 2013



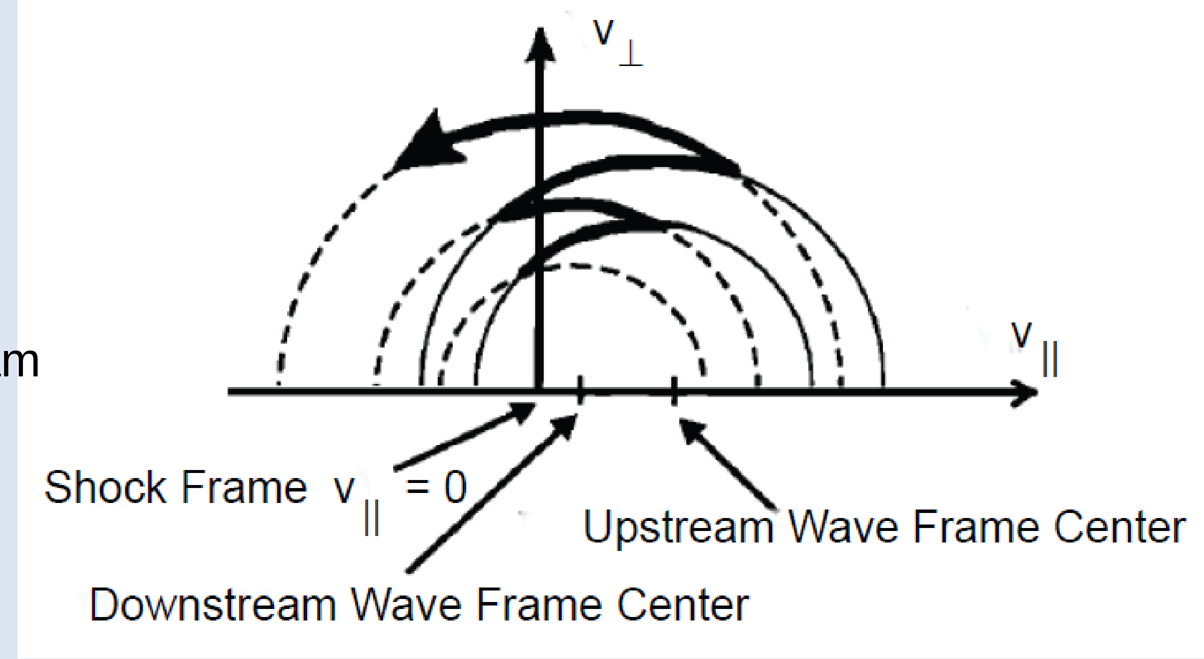
And many other works …

The Injection "Problem"

- Plasma simulations of parallel and quasi-parallel shocks

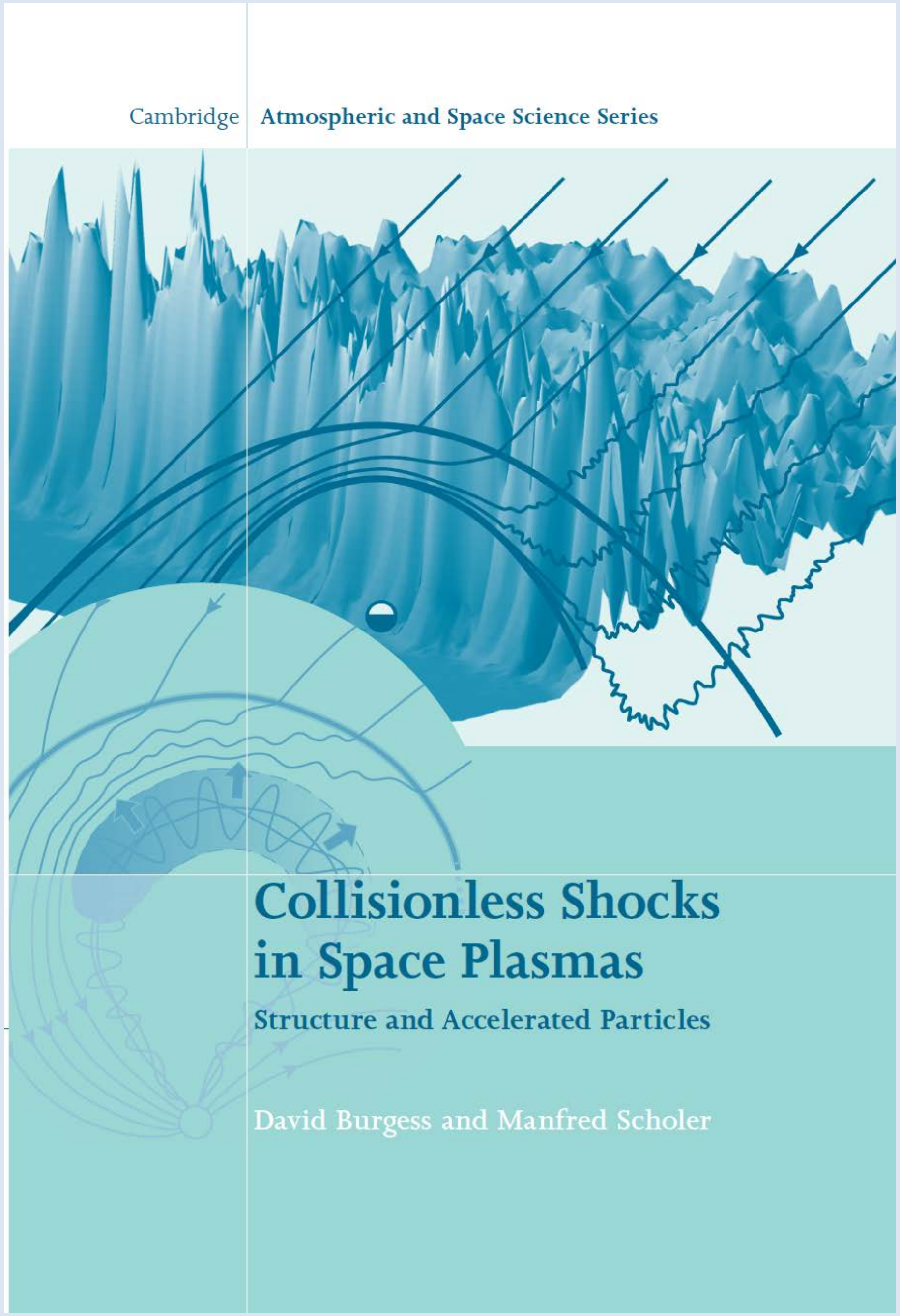
Can ions be extracted from the thermal population at the shock?

- Yes! **(What problem??)**
- Energized ions "reflected/ trapped" at shock transition
- No leakage from downstream heated population
- Development of simplified wave-shock trapping model (and other models)



Sugiyama and Terasawa 1999

For more details ….



Fin